

A Guide to Higher-Order Homophily

Moritz Laber, Brennan Klein



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NetSci2026, Boston, MA, USA June 3, 2026

Homophily & Heterophily

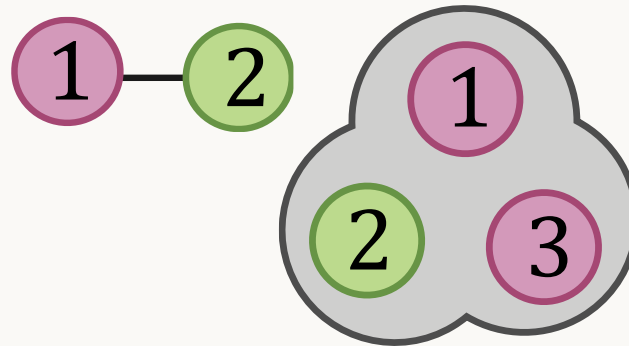
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Homophily is the above-chance occurrence of **interactions** between similar nodes.

Connectivity

(Hyper)edges indicating, e.g.,
co-authorship, discussions,
teams, ...

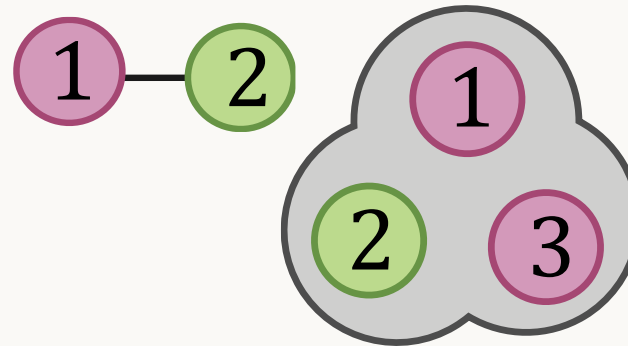


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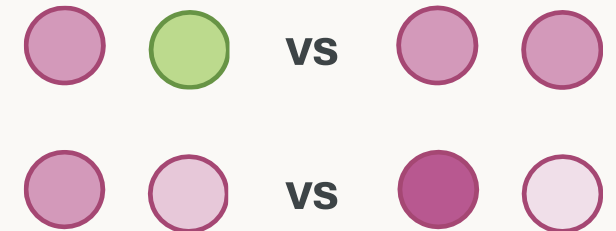
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Similarity

- *discrete attributes*: equality
- *continuous attributes*: inner products and distances



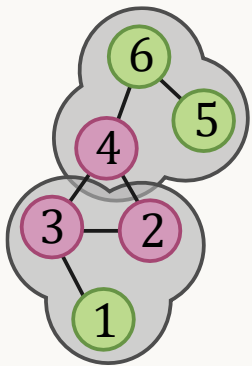
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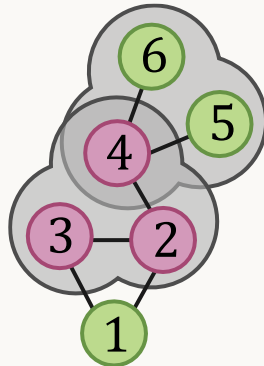
Null model

preserve relative class sizes,
and structural properties, e.g.,
average degree.

$\mathbb{P}(\mathcal{H}_1)$

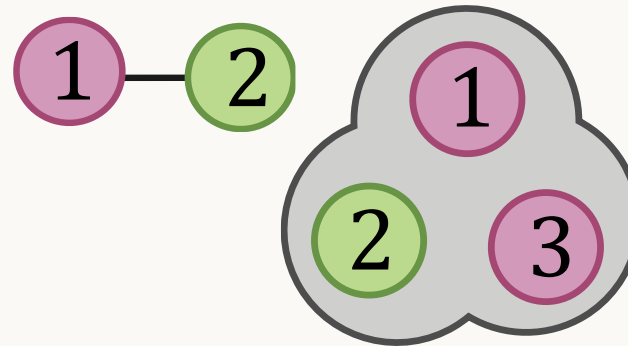


$\mathbb{P}(\mathcal{H}_2)$



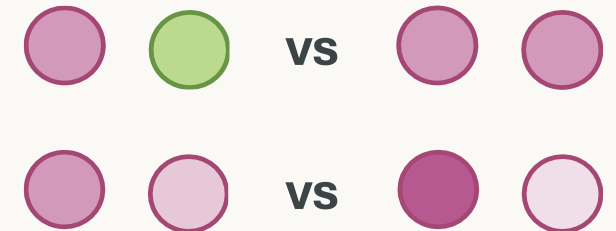
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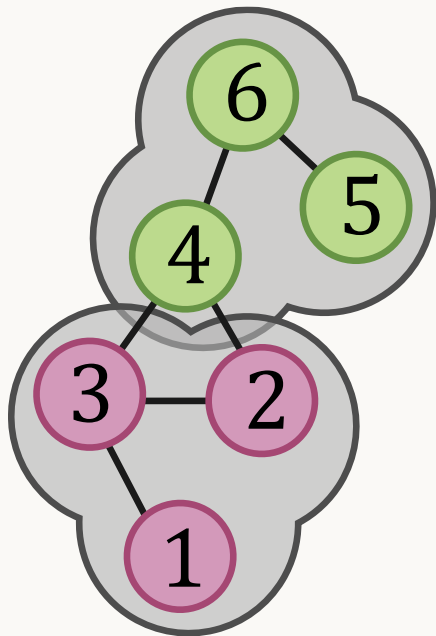
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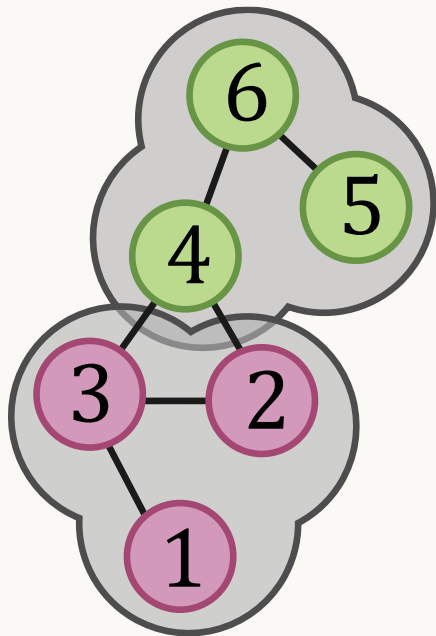


homophily

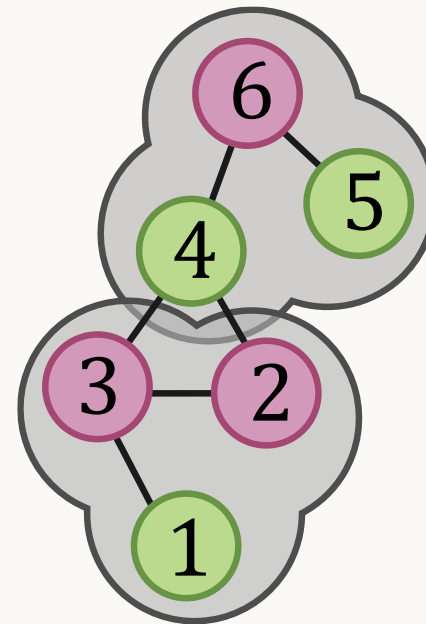
Homophily & Heterophily

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Heterophily is the above-chance occurrence of interactions between **dissimilar** nodes.



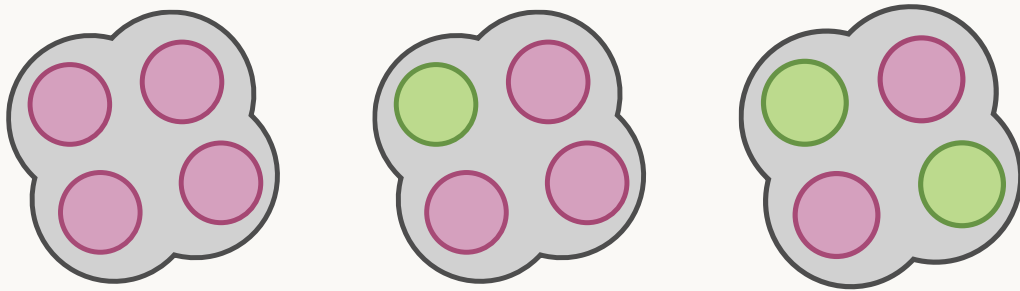
homophily



heterophily

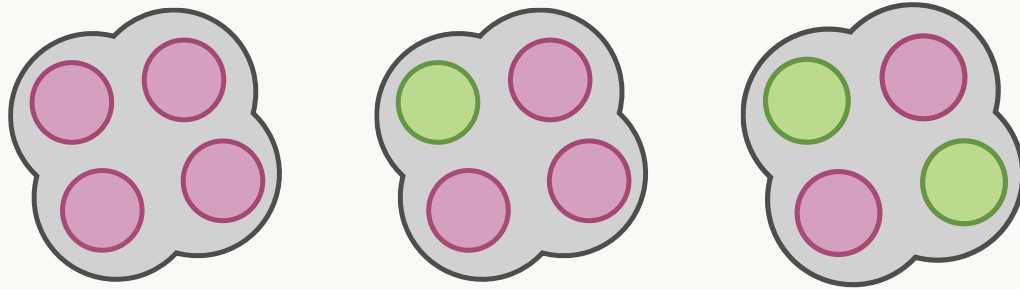
Higher-Order Homophily

Do **within-class** hyperedges make sense?

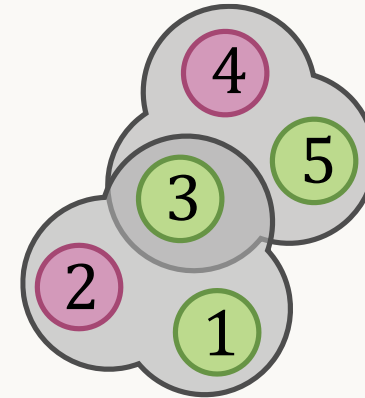


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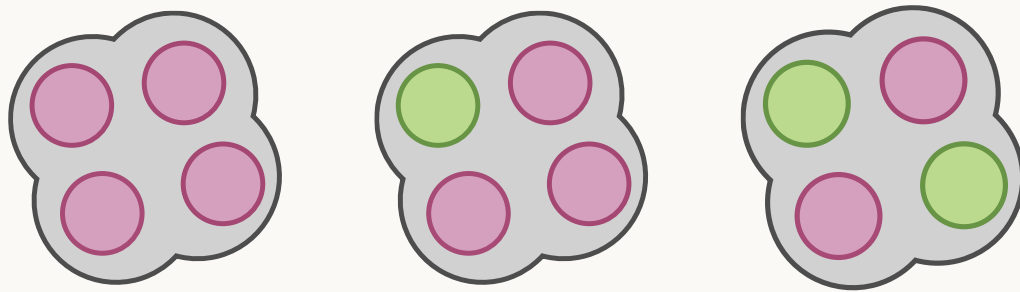


There are combinatorial **constraints**.

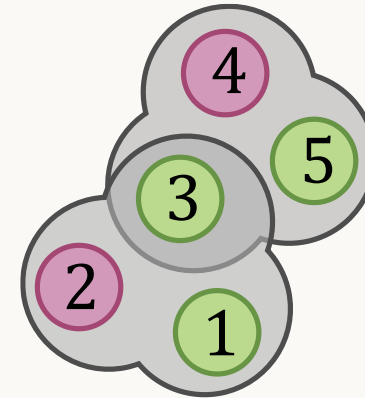


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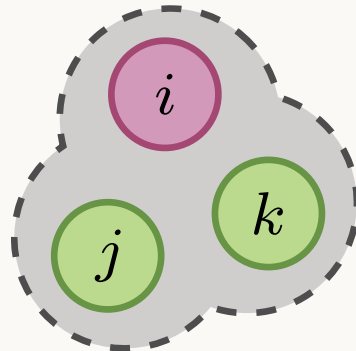
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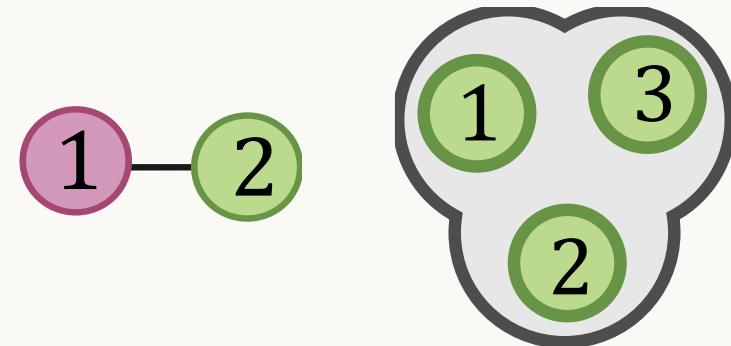
There are **a lot** of possible hyperedges.

$$E_{n,\alpha}^{\max} = \binom{n}{\alpha}$$

$$\frac{E_{1000,4}^{\max}}{E_{1000,2}^{\max}} \approx 8 \cdot 10^4$$

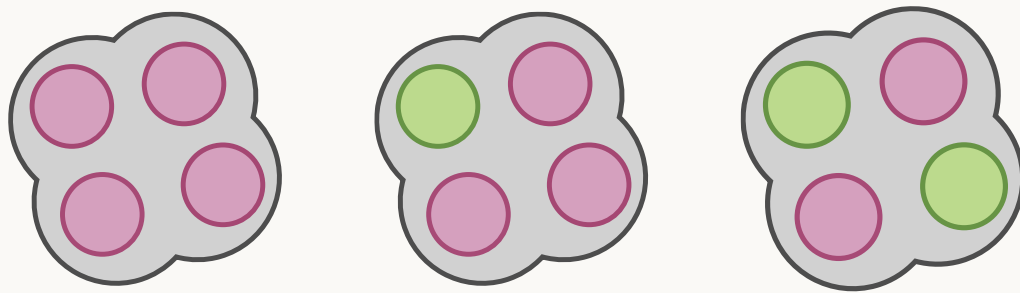


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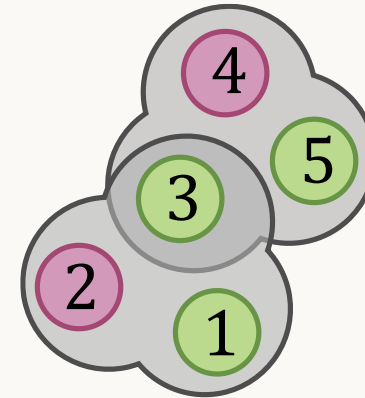


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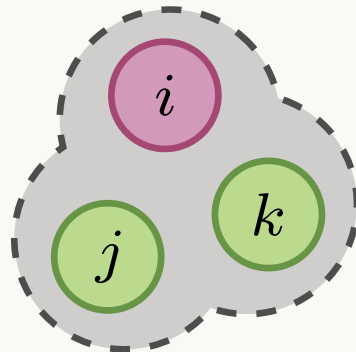
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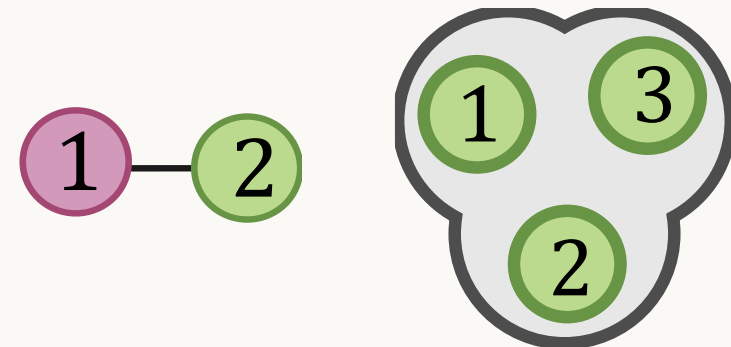
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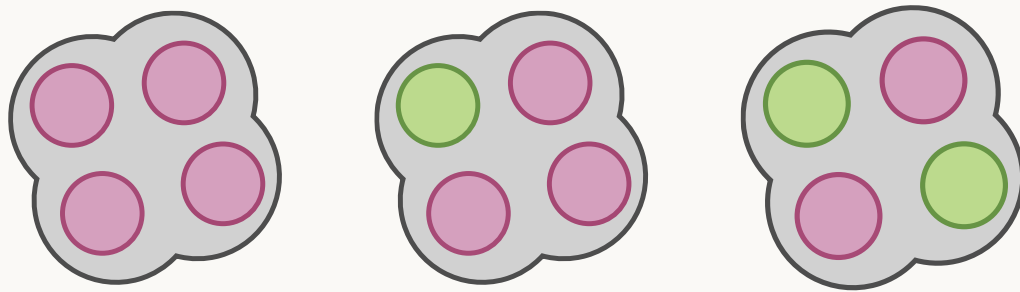


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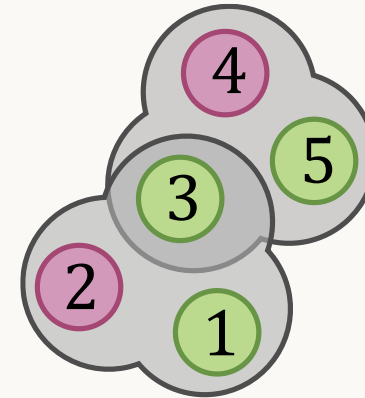


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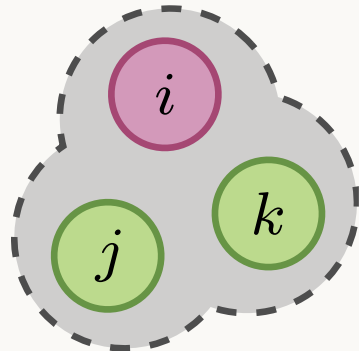
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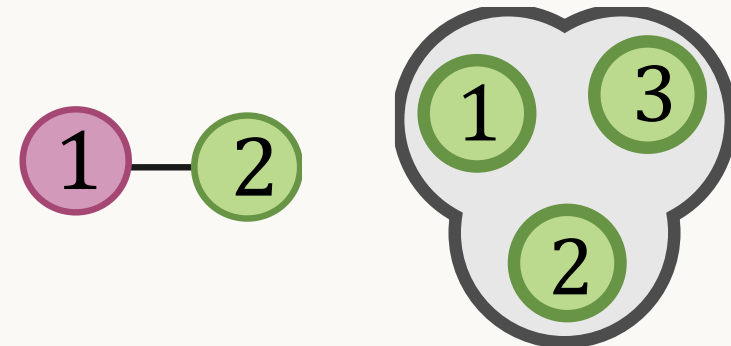
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A Guide to Higher-Order Homophily

Measures

Models

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Measures

- We survey different **definitions**.
 - Hypergraph homophily
 - Simplicial homophily
 - Perplexity-Homophily index
 - Random Walk HyperSegregation
 - Message Passing Homophily
 - Hypergraph assortativity
 - Hypergraph Modularity
 - Clique Projection

RESEARCH ARTICLE | NETWORK SCIENCE

Combinatorial characterizations and impossibilities for higher-order homophily

NATE VELDT, AUSTIN R. BENSON, AND JON KLEINBERG [Authors Info](#)

SCIENCE ADVANCES • 6 Jan 2023 • Vol 9, Issue 1 • DOI:10.1126/sciadv.abg3131

RESEARCH ARTICLE | APPLIED PHYSICAL SCIENCES

Higher-order homophily on simplicial complexes

Arnab Sarkar, Natalie J. ... Edited by Matthew Jackson, Stanford

March 12, 2024 | 121 (121) e7

ARTICLE

Beyond Boundaries: Capturing Social Segregation on Hypernetworks

Authors: Andrea Faila, Giulio Rossetti, Francesco Cauteruccio [Authors Info & Claims](#)

Social Networks Analysis and Mining: 16th In Pages 40 - 55 • https://doi.org/10.1007/978-3-031-11111-2_3

Published: 02 September 2024 [Publication Info](#)

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Hypergraph assortativity: A dynamical systems perspective

etworks with Higher-Order Interactions

G. Restrepo

JOURNAL ARTICLE

Modularity based community detection in hypergraphs

Bogumił Kamiński, Paweł Misiorek, Paweł Prałat, François Théberge

Journal of Complex Networks, Volume 12, Issue 5, October 2024, cnae041, <https://doi.org/10.1093/comnet/cnae041>

Published: 21 October 2024 [Article history](#)

JOURNAL ARTICLE

Configuration models of random hypergraphs

Philip S Chodrow

Journal of Complex Networks, Volume 8, Issue 3, June 2020, cnaa018, <https://doi.org/10.1093/comnet/cnaa018>

RESEARCH ARTICLE | NETWORK SCIENCE

Generative hypergraph clustering: From blockmodels to modularity

PHILIP S. CHODROW, NATE VELDT, AND AUSTIN R. BENSON [Authors Info & Affiliations](#)

SCIENCE ADVANCES • 7 Jul 2021 • Vol 7, Issue 28 • DOI:10.1126/sciadv.abh1303

RESEARCH ARTICLE | NETWORK SCIENCE

Perplexity-Homophily Index: Homophily through Diversity in Hypergraphs

Chandrakala Meena
IISER Pune
India
chandrakala@iiserpune.ac.in

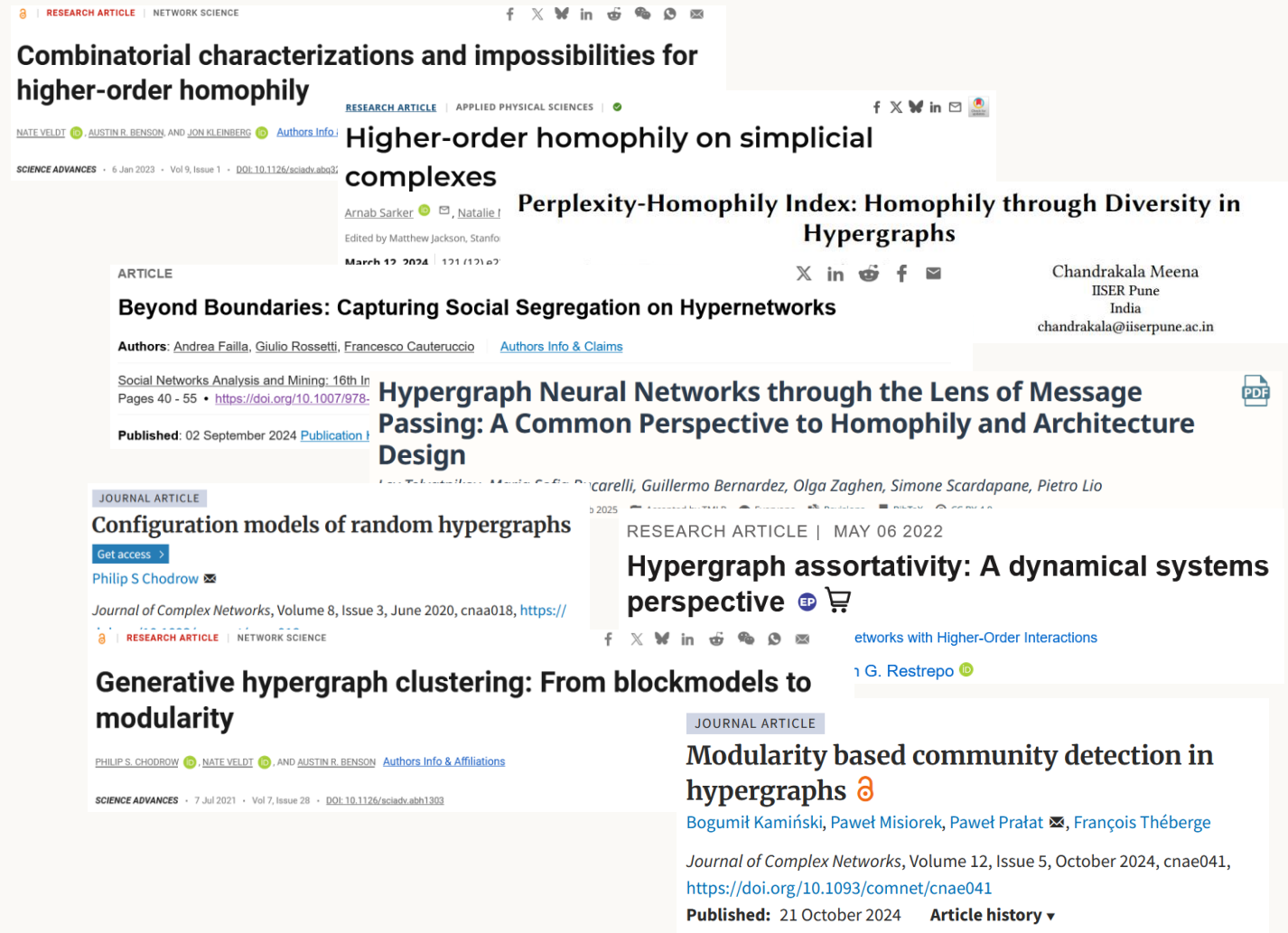
Hypergraph Neural Networks through the Lens of Message Passing: A Common Perspective to Homophily and Architecture Design

Guillaume Carlier, Guillermo Bernardez, Olga Zaghen, Simone Scardapane, Pietro Lio

A Guide to Higher-Order Homophily

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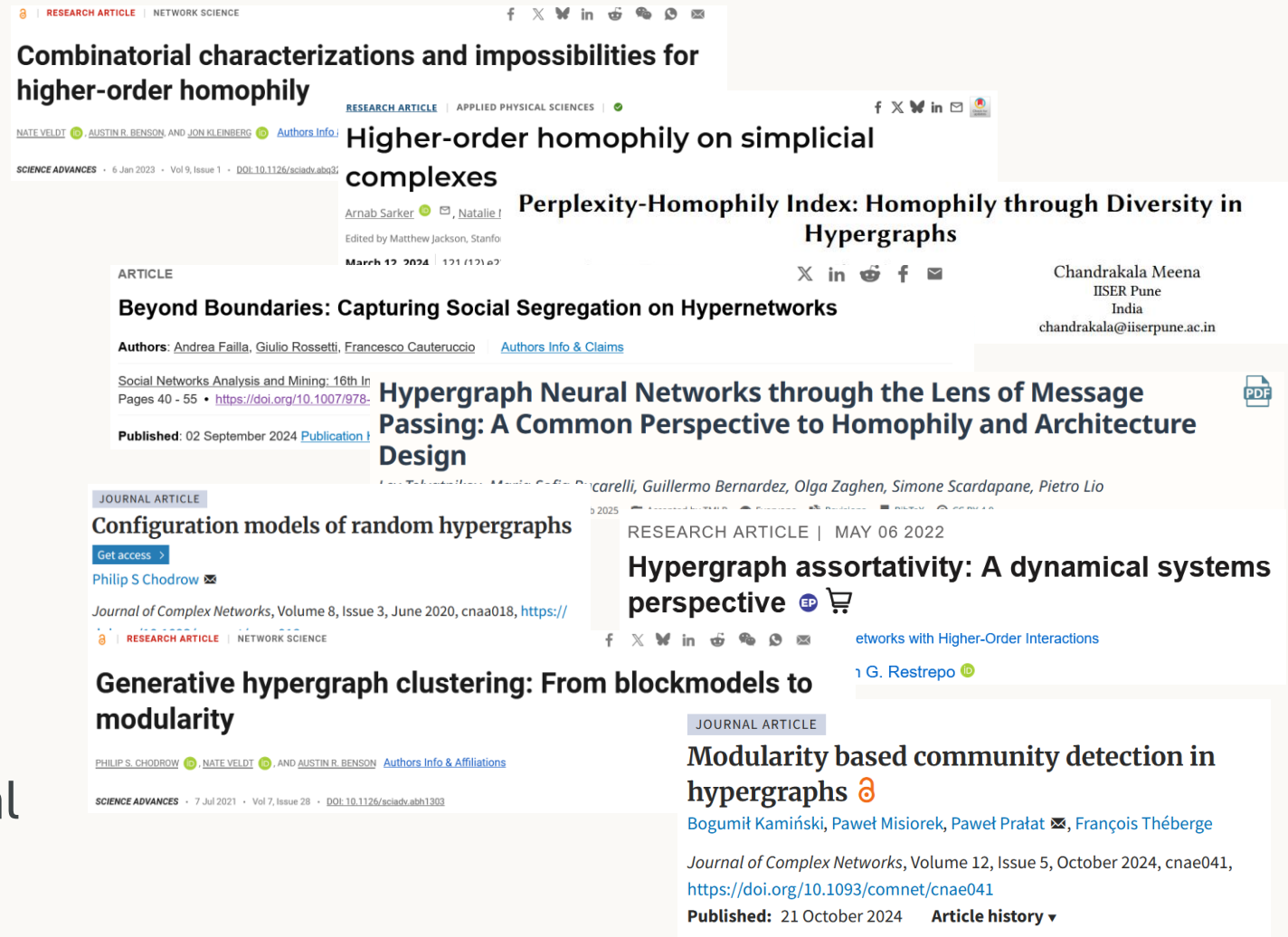
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 - Clique Projection
- We provide didactic **examples**.



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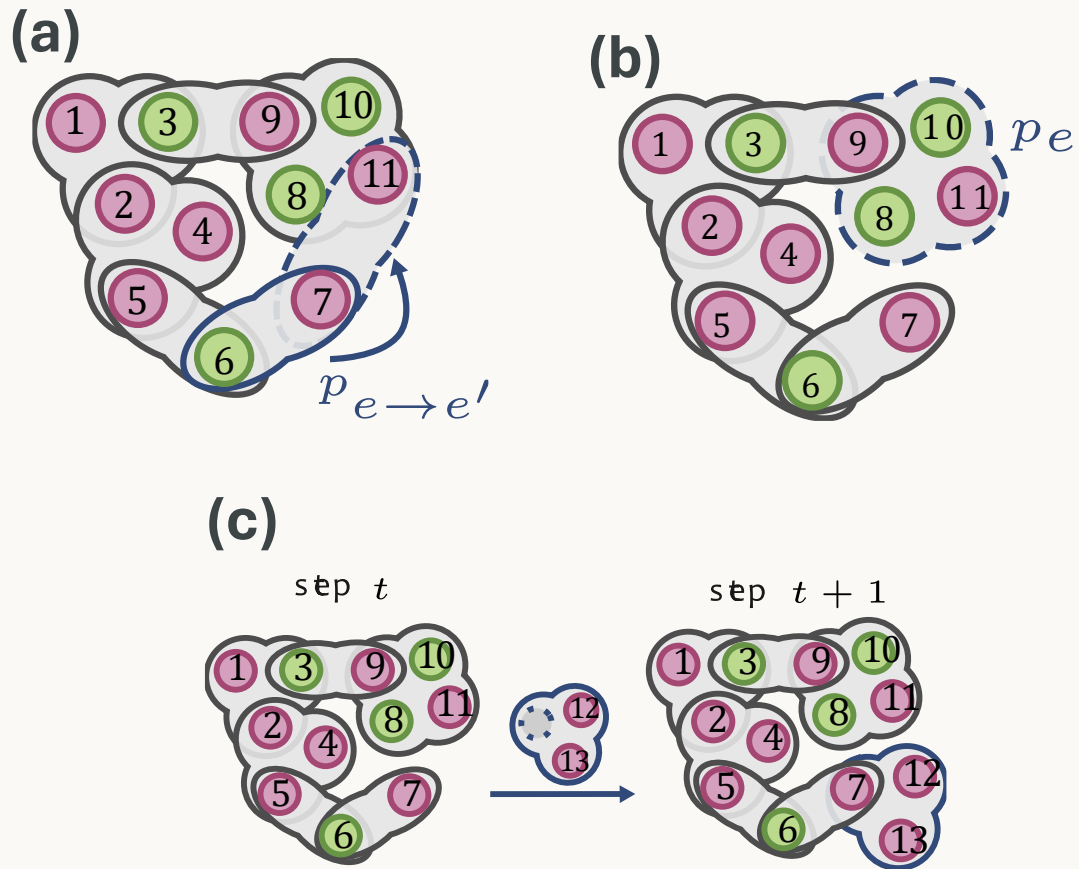
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- We provide didactic **examples**.
- We **provide guidance** on conceptual insights, limitations, challenges...



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$\mathbb{P}(\mathcal{H})$

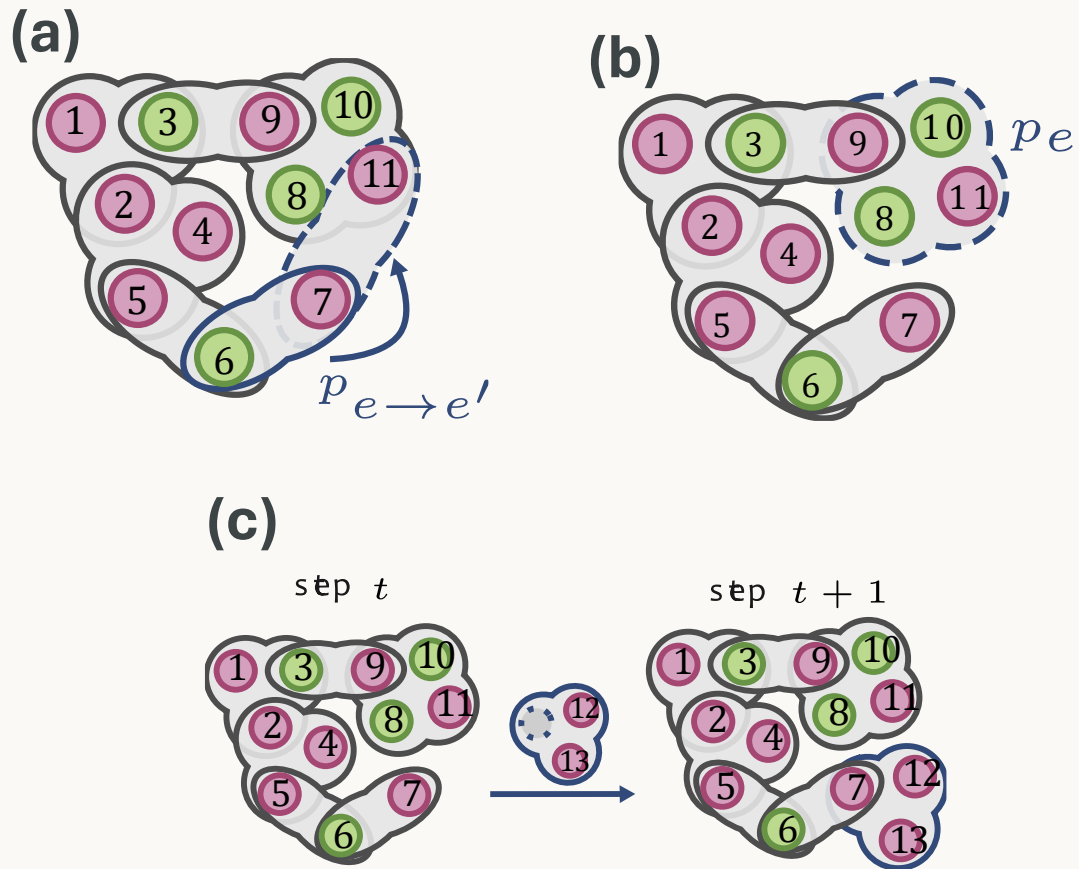


Models

- We provide an **overview of different classes** of models.
 - Null models
 - Hypergraph SBMs
 - Geometric models
 - Growing / Mechanistic models

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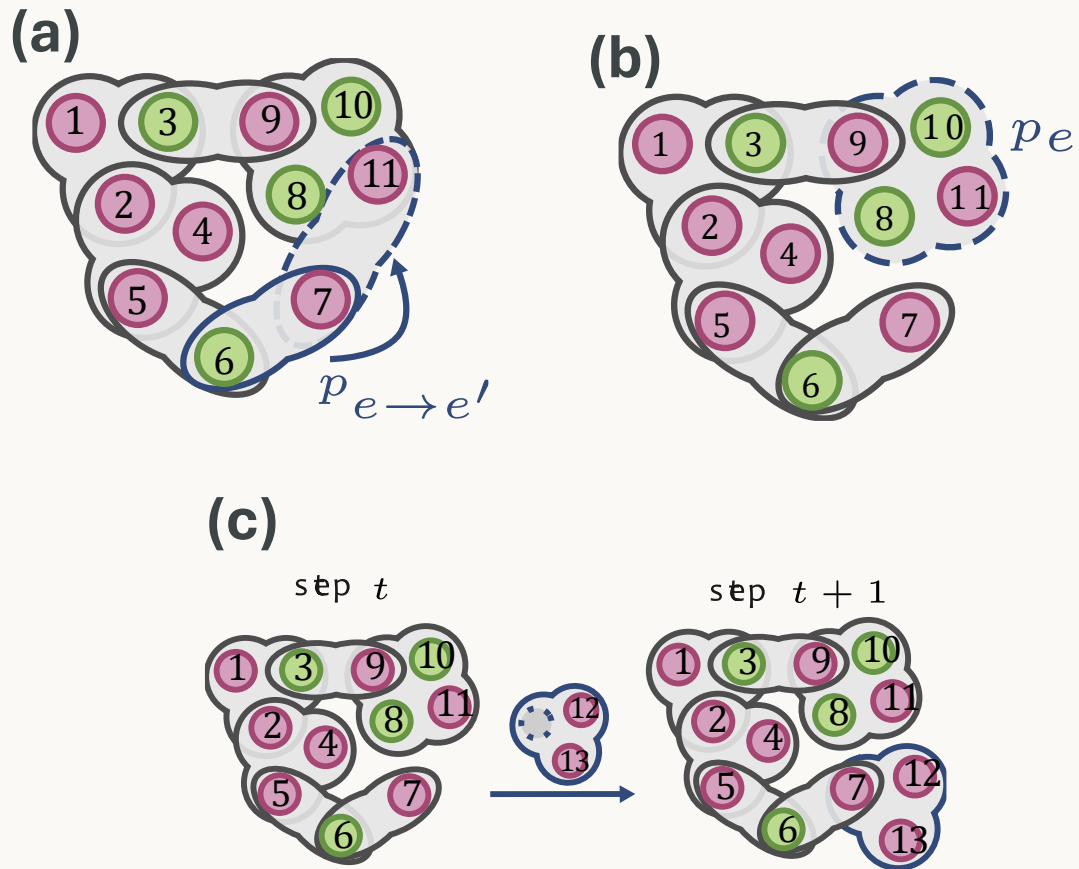


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- **Discuss their role** in for studying higher-order homophily.

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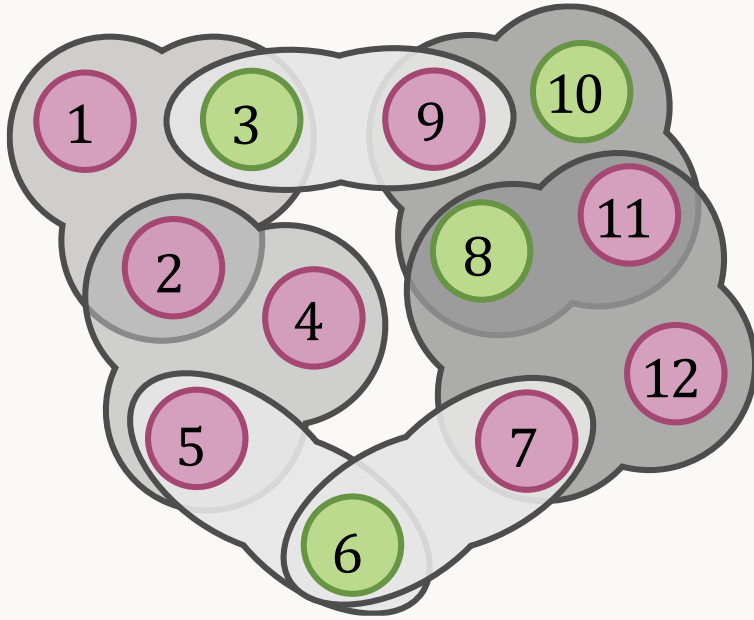
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 - Null models
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- Discuss their role in for studying higher-order homophily.
- Highlight **open questions**.

Example Measure: Hypergraph Homophily



Properties

$N = 12$ number of nodes

$c \in \{0, 1\}$ two classes

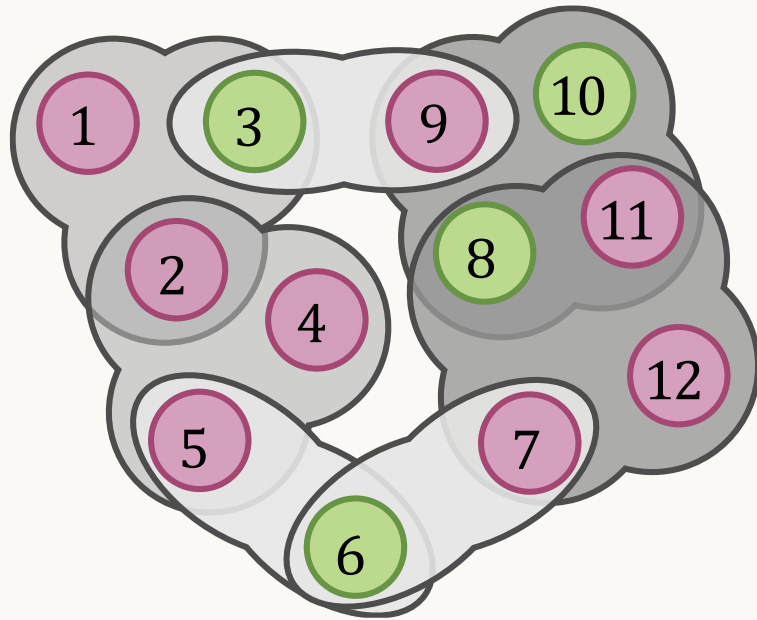
$N_0 = 8$ number of **class 0** nodes

$N_1 = 4$ number of **class 1** nodes

$E = 7$ number of hyperedges

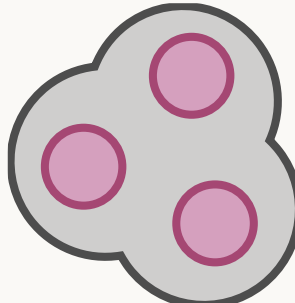
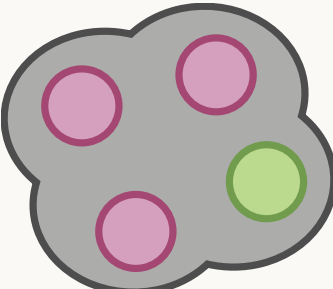
$\alpha \in \{2, 3, 4\}$ hyperedge sizes

Example Measure: Hypergraph Homophily

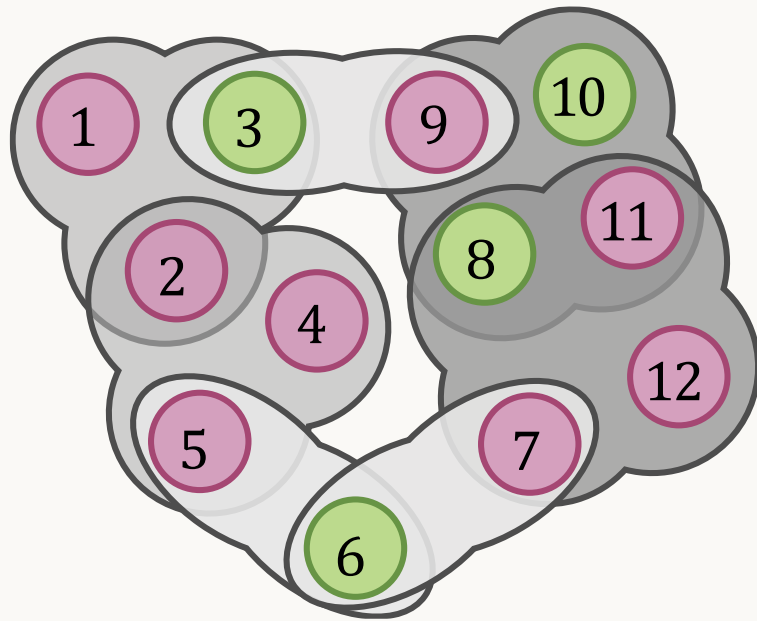


Number of hyperedges of type

$$E_{\alpha,t}^{(c)} = |\{e \in E : t_c(e) = t \wedge |e| = \alpha\}|$$

	Size	Type
	$\alpha = 2$	$t_0(e) = 1$ $t_1(e) = 1$
	$\alpha = 3$	$t_0(e) = 3$ $t_1(e) = 0$
	$\alpha = 4$	$t_0(e) = 3$ $t_1(e) = 1$

Example Measure: Hypergraph Homophily



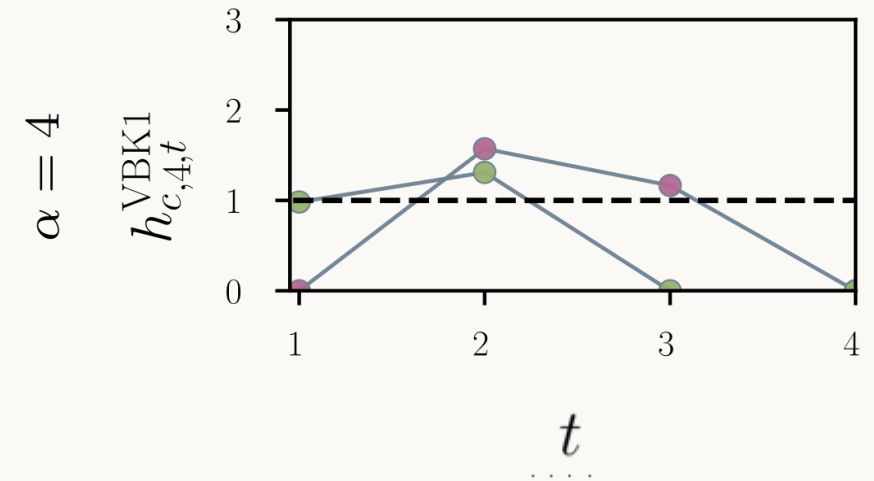
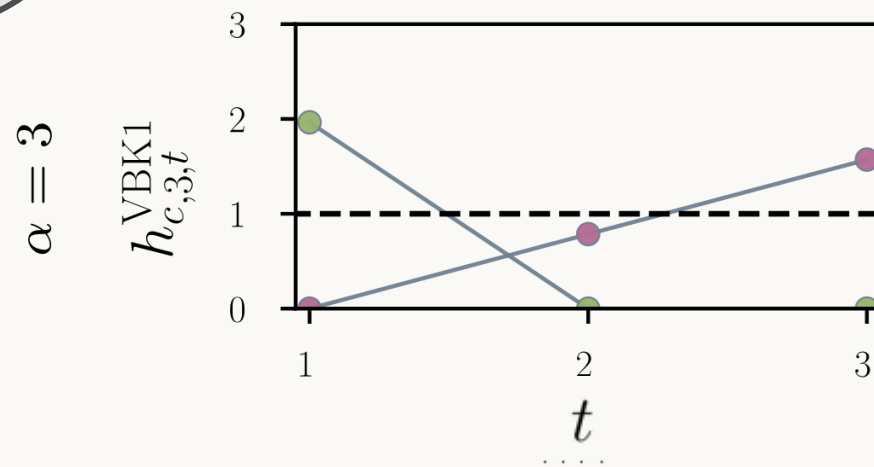
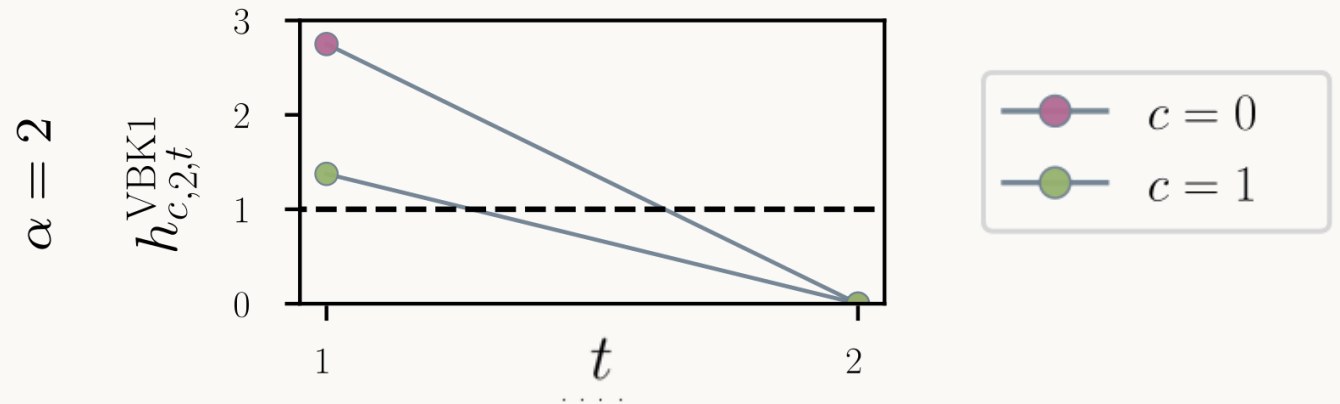
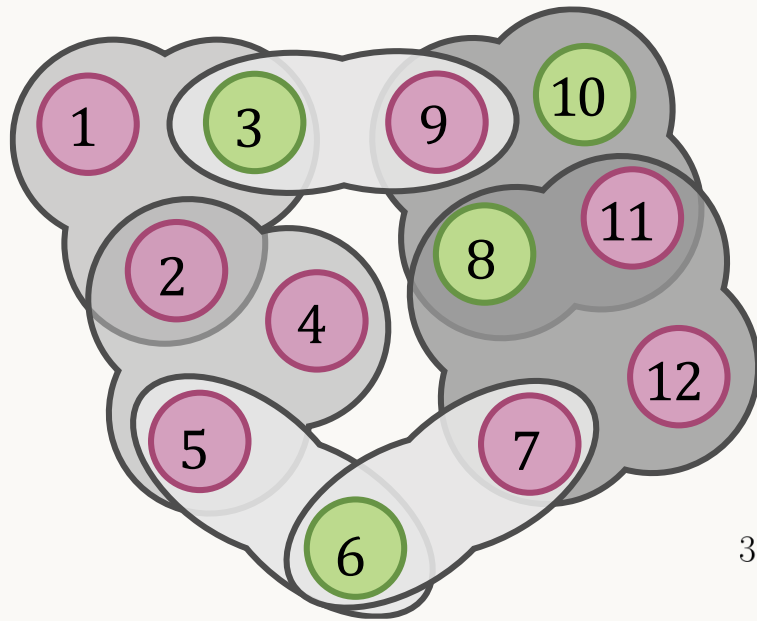
$$a_{c,\alpha,t}^{\text{VBK1}} = \frac{tE_{\alpha,t}^{(c)}}{\sum_{t'=1}^{\alpha} t'E_{\alpha,t'}^{(c)}}$$

$$b_{c,\alpha,t}^{\text{VBK1}} = \frac{\binom{N_c-1}{t-1} \binom{N-N_c}{\alpha-t}}{\binom{N-1}{\alpha-1}}$$

$$h_{c,\alpha,t}^{\text{VBK1}} = \frac{a_{c,\alpha,t}^{\text{VBK1}}}{b_{c,\alpha,t}^{\text{VBK1}}}$$

c	α	t	$E_{\alpha,t}^{(c)}$	$a_{c,\alpha,t}^{\text{VBK1}}$	$b_{c,\alpha,t}^{\text{VBK1}}$	$h_{c,\alpha,t}^{\text{VBK1}}$
0	2	0	0	—	—	—
		1	3	3/3	4/11	11/4
		2	0	0/3	7/11	0
	3	0	0	—	—	—
		1	0	0/5	6/55	0
		2	1	2/5	28/55	11/14
		3	1	3/5	21/55	22/14
	4	0	0	—	—	—
		1	0	0/5	4/165	0
		2	1	2/5	42/165	33/21
		3	1	3/5	85/165	99/85
		4	0	0/5	35/165	0
1	2	0	0	—	—	—
		1	3	3/3	8/11	11/8
		2	0	0/3	3/11	0
	3	0	0	—	—	—
		1	1	1/1	28/55	55/28
		2	0	0/1	24/55	0
		3	0	0/1	3/55	0
	4	0	0	—	—	—
		1	1	1/3	56/165	55/56
		2	1	2/3	84/165	55/42
		3	0	0/3	24/165	0
		4	0	0/3	1/165	0

Example Measure: Hypergraph Homophily



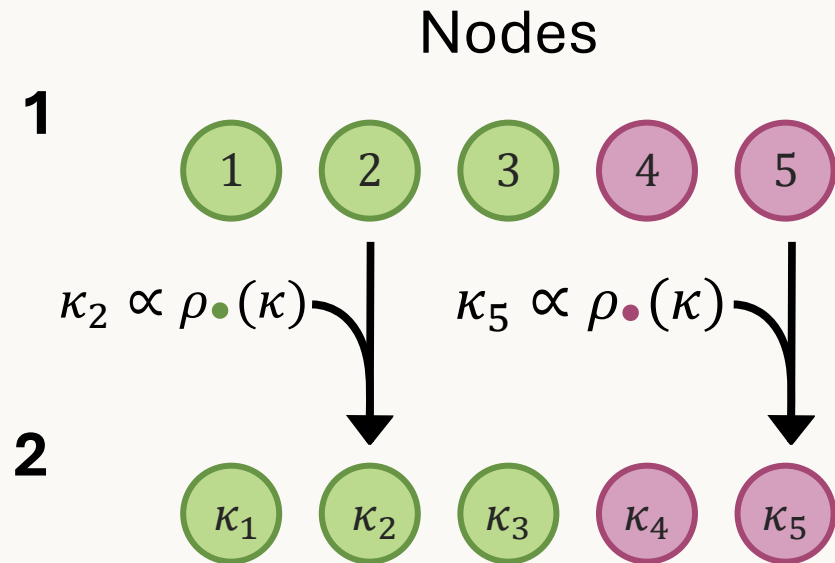
Example Model: H3 Model

Nodes

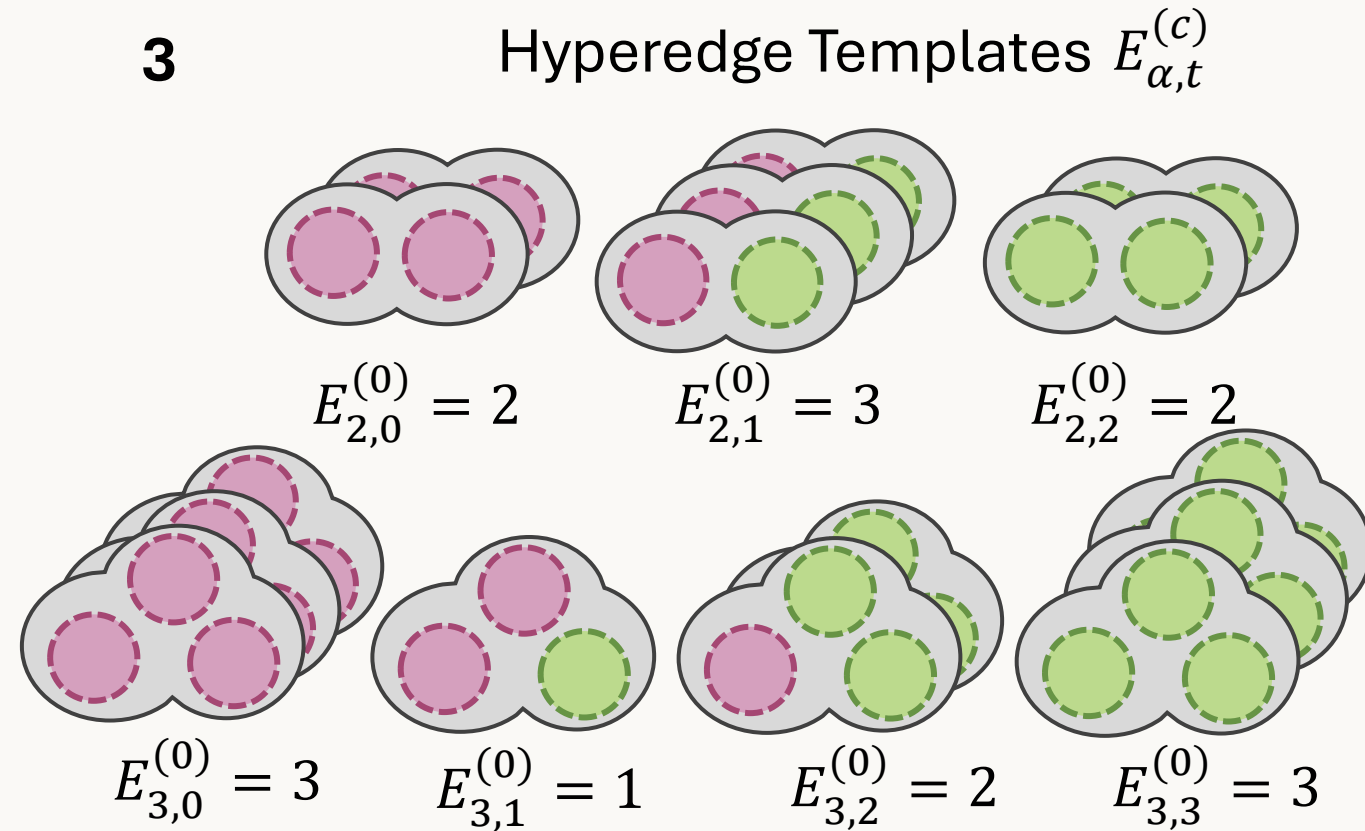
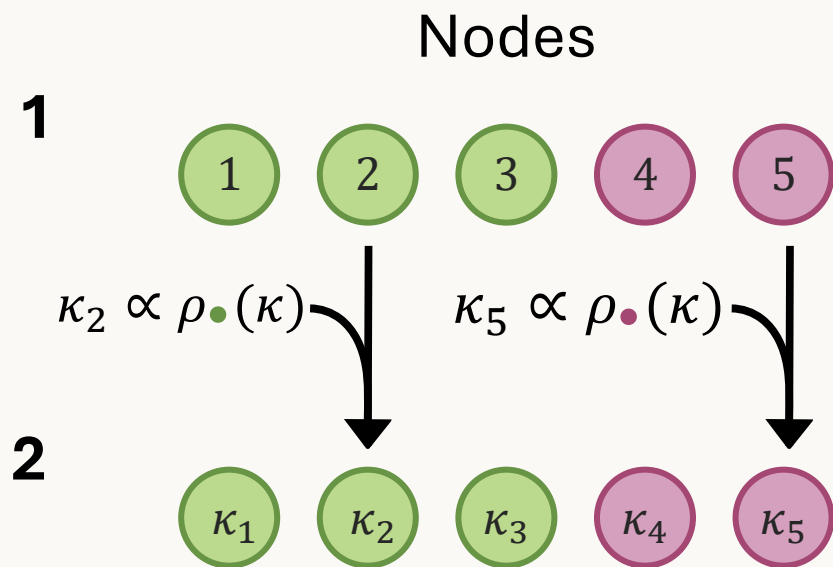
1



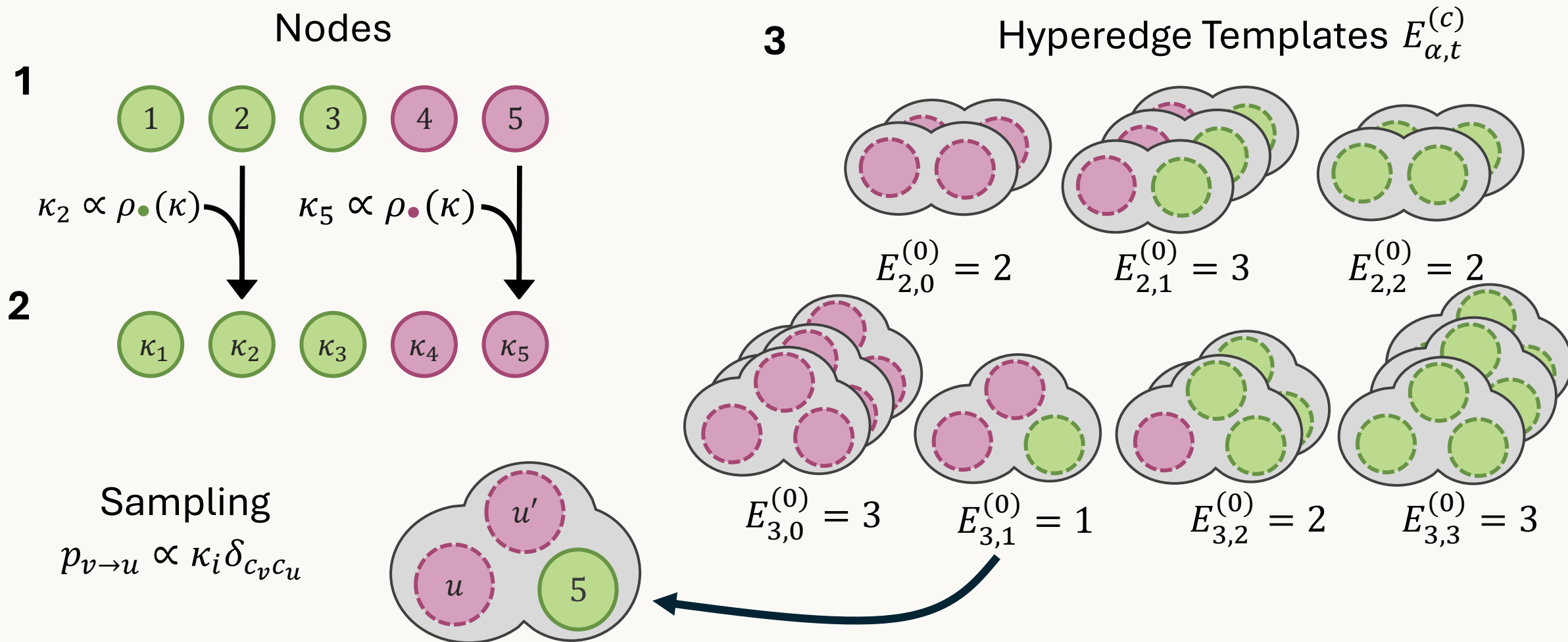
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Laber, Dies, Ehlert et al. Comm. Phys. **9** 13 (2026)

Conclusion

Key Insights

- Hyperedges are **not either within- or between-class**. Higher-order mixing patterns are much richer.
- Mixing patterns can **vary with hyperedge size**.
- There is a **lot of methods** out there, but less effort to re-connect theory in the social sciences and a lack of synthesis.

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- What is the “right” **scale** for measuring homophily?
- How should we **summarize the high-dimensional information** that attributed hypergraphs provide?
- How do we create hypergraph **models that are principled and computationally efficient**?
- **Do higher-order mixing patterns matter** beyond pairwise affinity?

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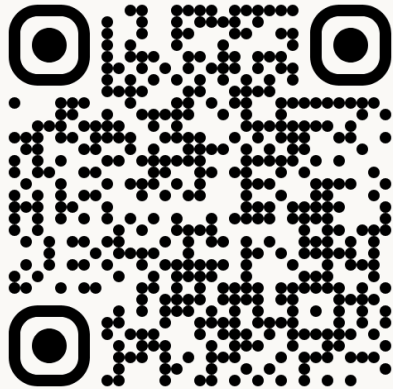
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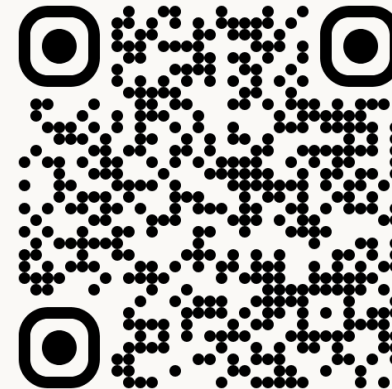
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<https://arxiv.org/abs/2606.02537v1>



Effects of higher-order interactions and homophily on information access inequality

<https://doi.org/10.1038/s42005-025-02445-y>



Other Projects

Erdős-Rényi Random Hypergraphs (Thursday, Higher-Order & Multi-Layer Networks 3)

Percolation in Growing Stochastic Block Model (Today, Poster 129)

Robustness & Generalization in Uncertainty-aware MPNN (Today, Poster 39)

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