

Mixing Patterns in Higher-Order Networks: A Review

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IC²S², Burlington, VT, USA July 31, 2026

Homophily & Heterophily

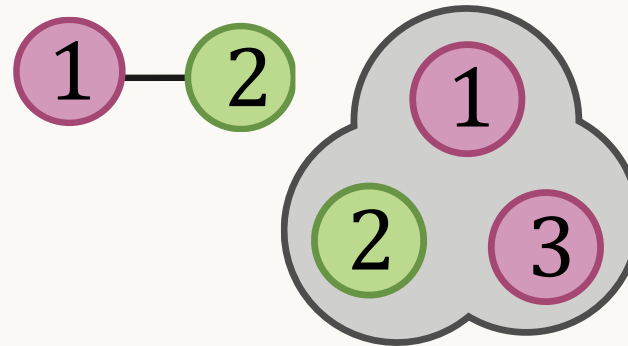
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Connectivity

(Hyper)edges indicating, e.g.,
co-authorship, discussions,
teams, ...

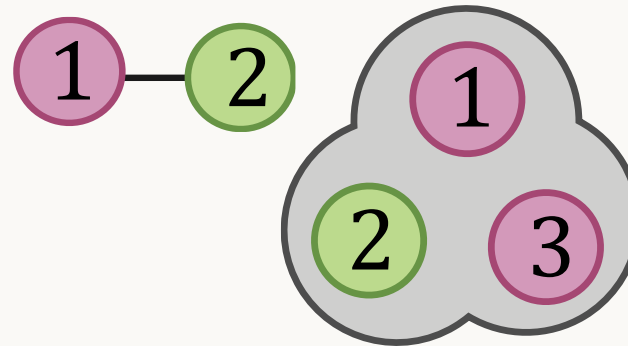


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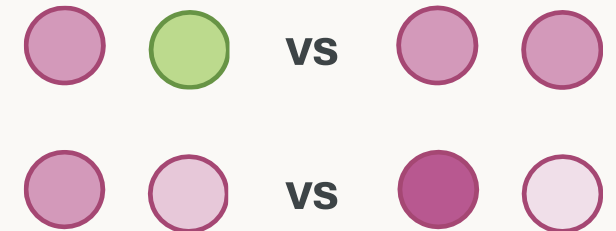
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Similarity

- *discrete attributes*: equality
- *continuous attributes*: inner products and distances

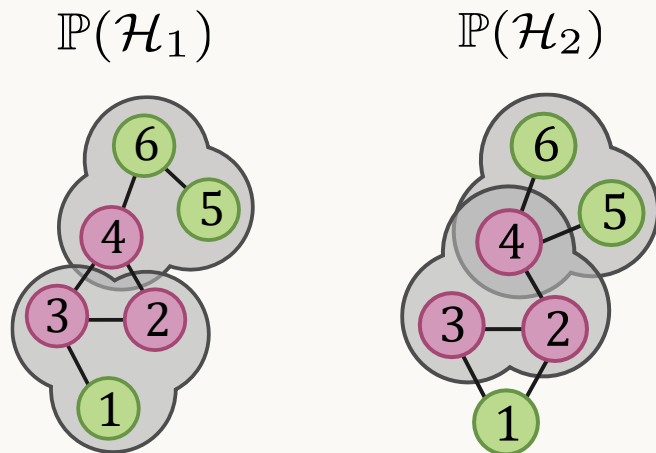


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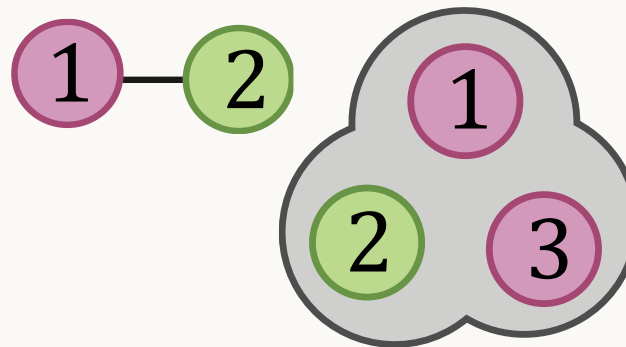
Null model

preserve relative class sizes,
and structural properties, e.g.,
average degree.



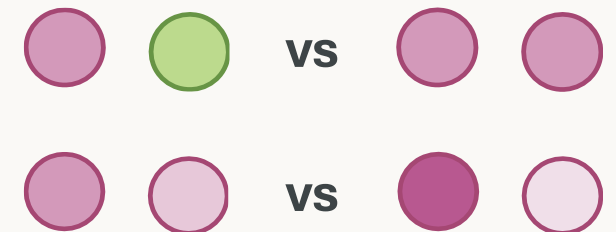
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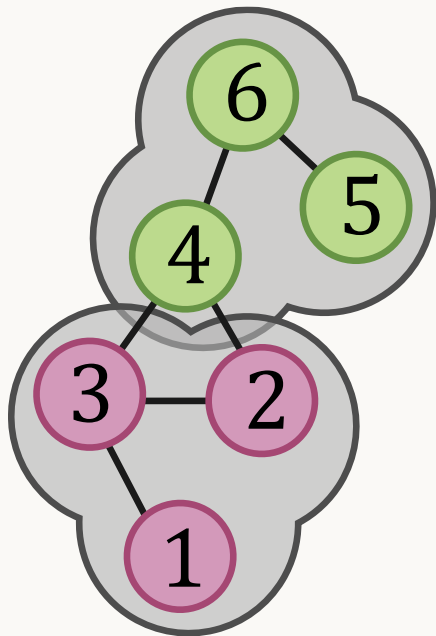
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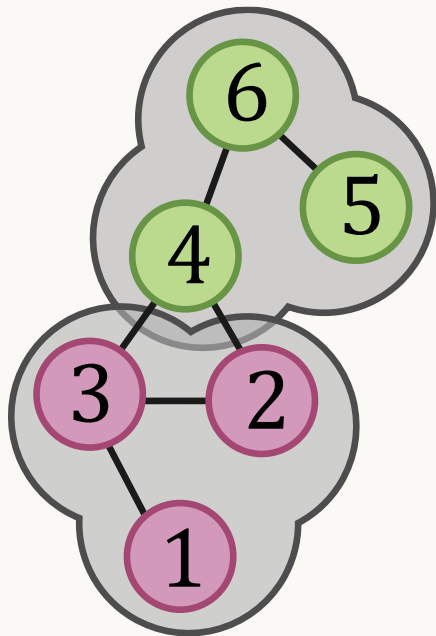


homophily

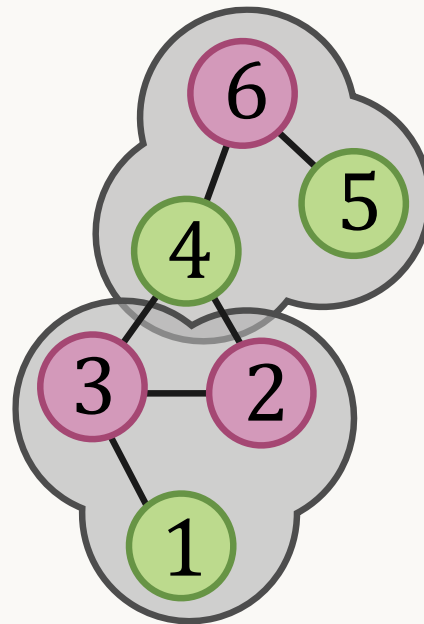
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Heterophily is the above-chance occurrence of interactions between **dissimilar** nodes.



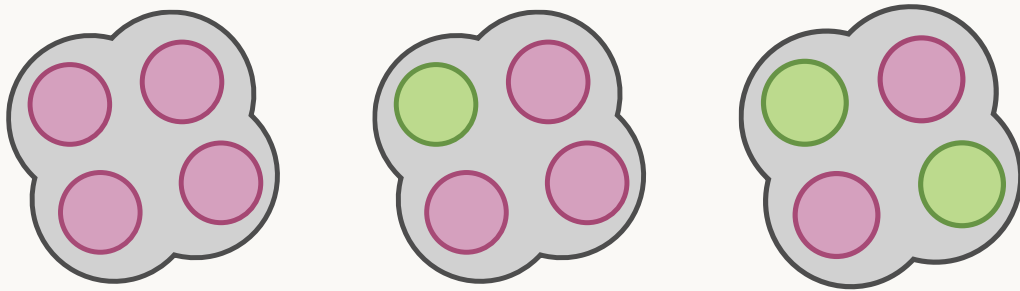
homophily



heterophily

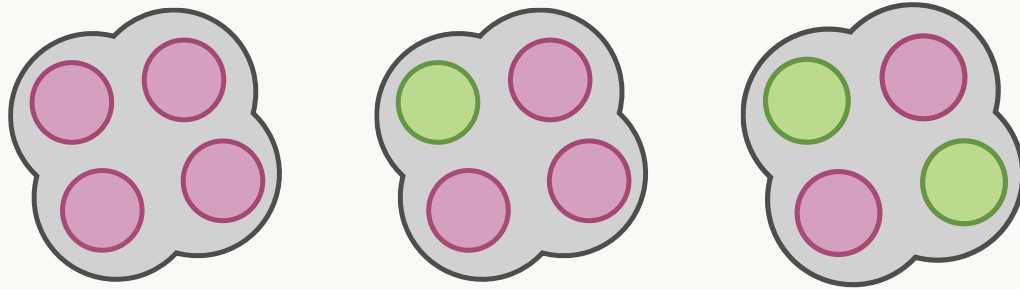
Higher-Order Homophily

Do **within-class** hyperedges make sense?

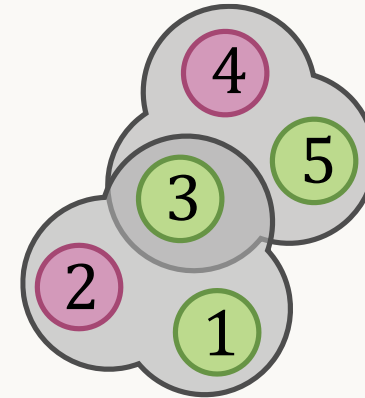


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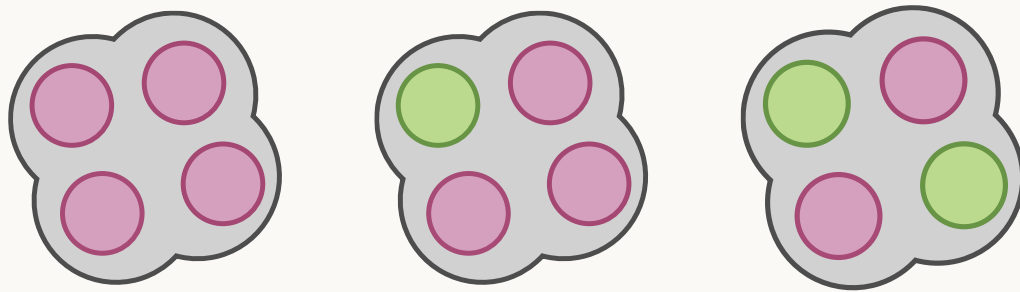


There are combinatorial **constraints**.

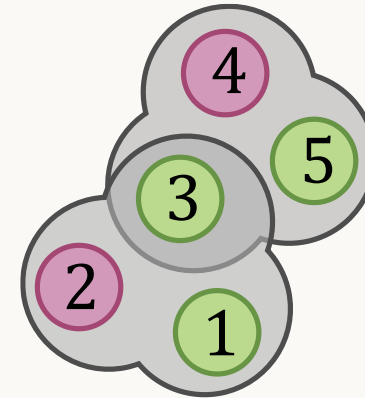


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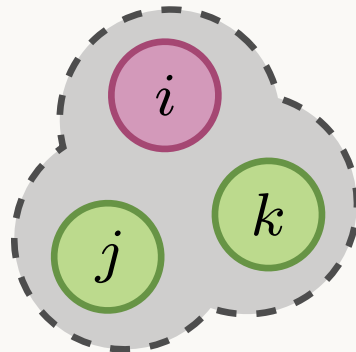
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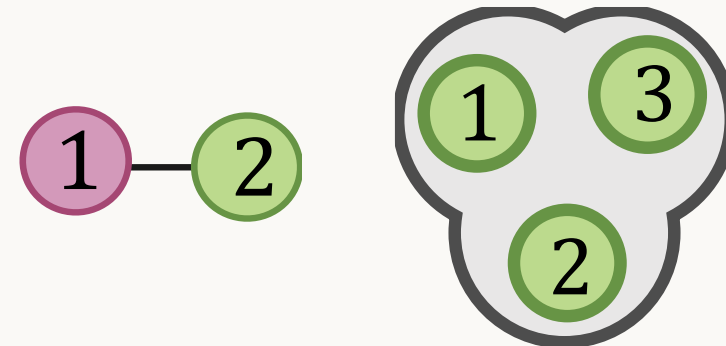
There are **a lot** of possible hyperedges.

$$E_{n,\alpha}^{\max} = \binom{n}{\alpha}$$

$$\frac{E_{1000,4}^{\max}}{E_{1000,2}^{\max}} \approx 8 \cdot 10^4$$

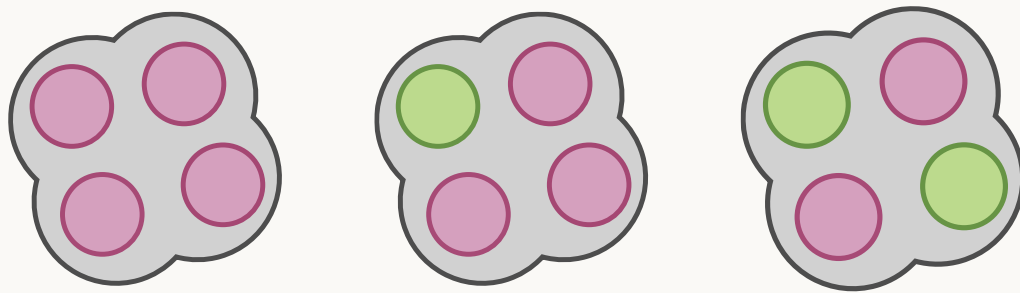


Mixing patterns can **vary by hyperedge size**.

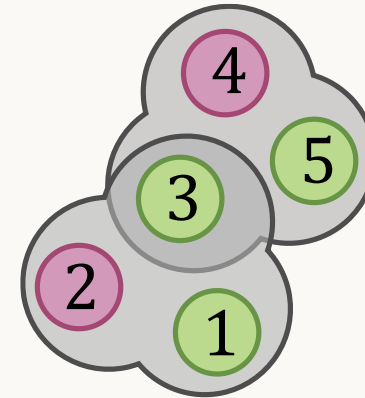


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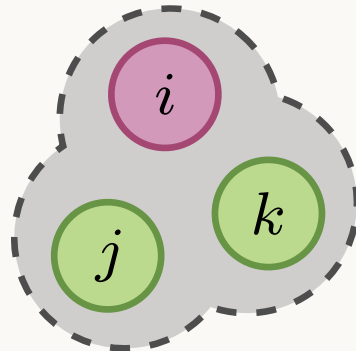
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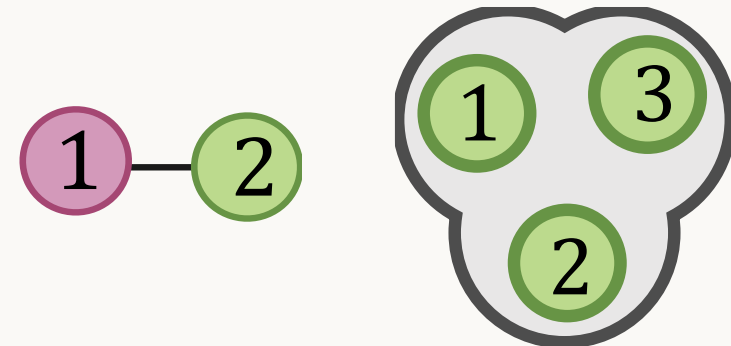
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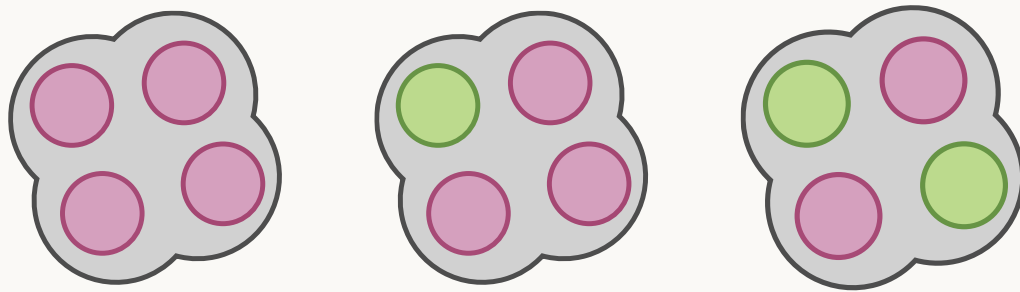


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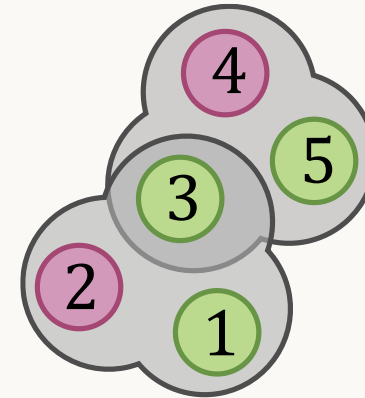


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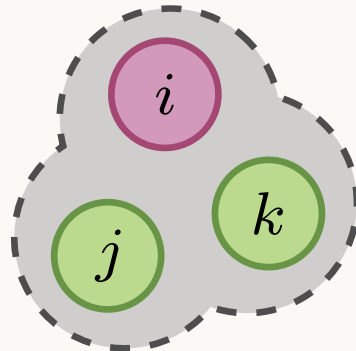
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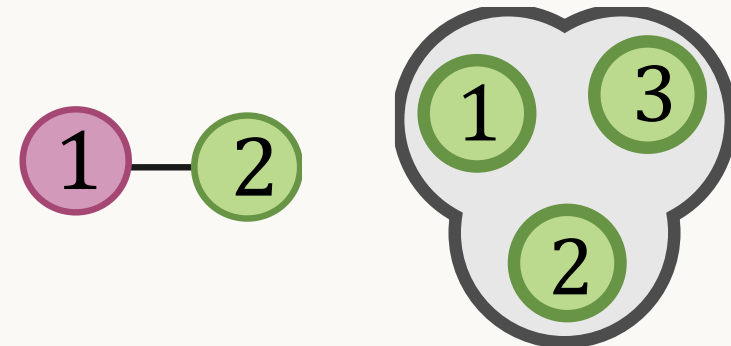
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Mixing patterns can **vary by hyperedge size**.



A Guide to Higher-Order Homophily

Measures

Models

A Guide to Higher-Order Homophily

Measures

- We survey different **definitions**.
 - Hypergraph homophily
 - Simplicial homophily
 - Perplexity-Homophily index
 - Random Walk HyperSegregation
 - Message Passing Homophily
 - Hypergraph assortativity
 - Hypergraph Modularity
 - Clique Projection

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NATE VELDT, AUSTIN R. BENSON, AND JON KLEINBERG [Authors Info](#)

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Chandrakala Meena
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etworks with Higher-Order Interactions

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Bogumił Kamiński, Paweł Misiorek, Paweł Prałat, François Théberge

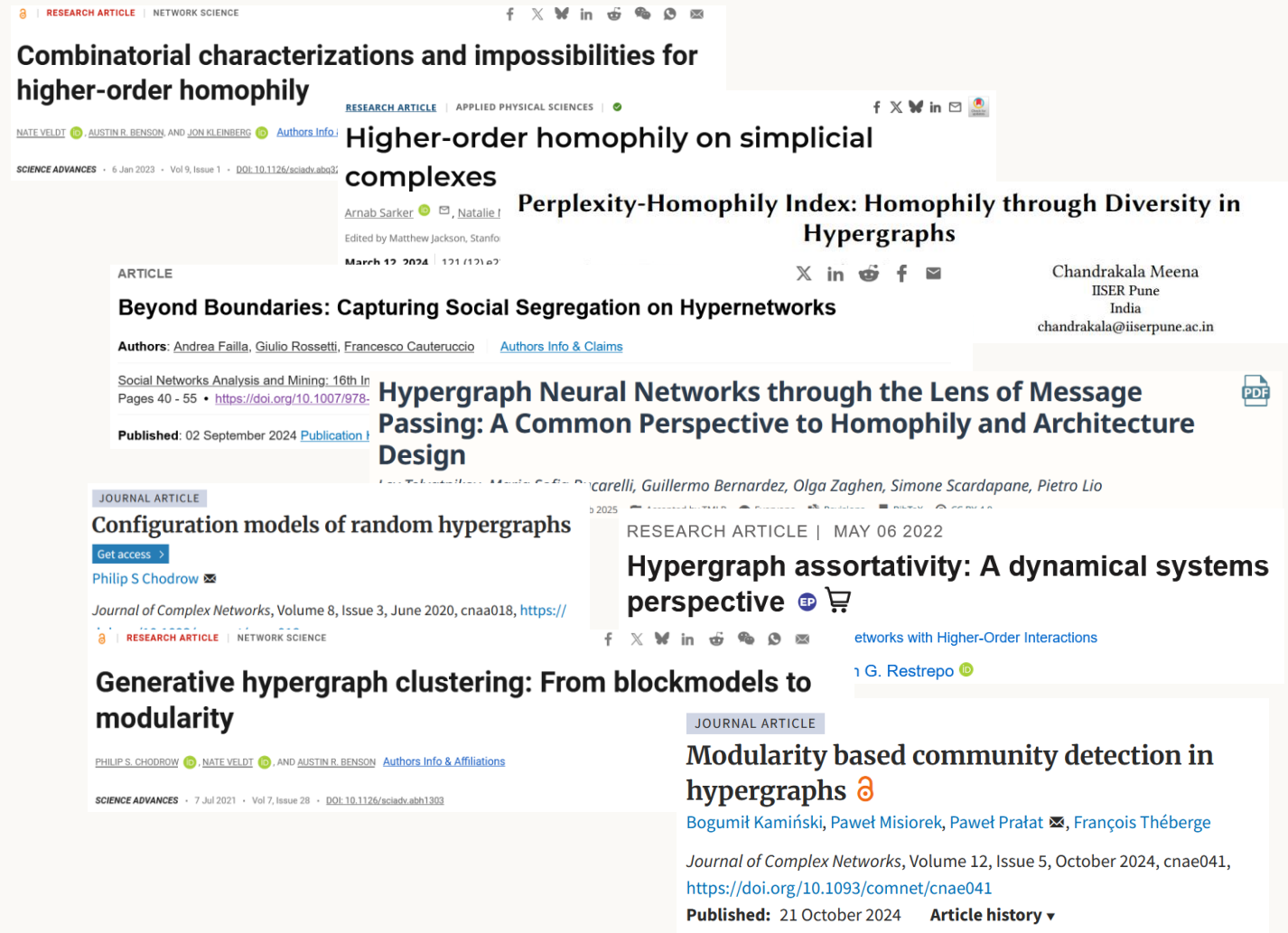
Journal of Complex Networks, Volume 12, Issue 5, October 2024, cnae041, <https://doi.org/10.1093/comnet/cnae041>

Published: 21 October 2024 [Article history](#)

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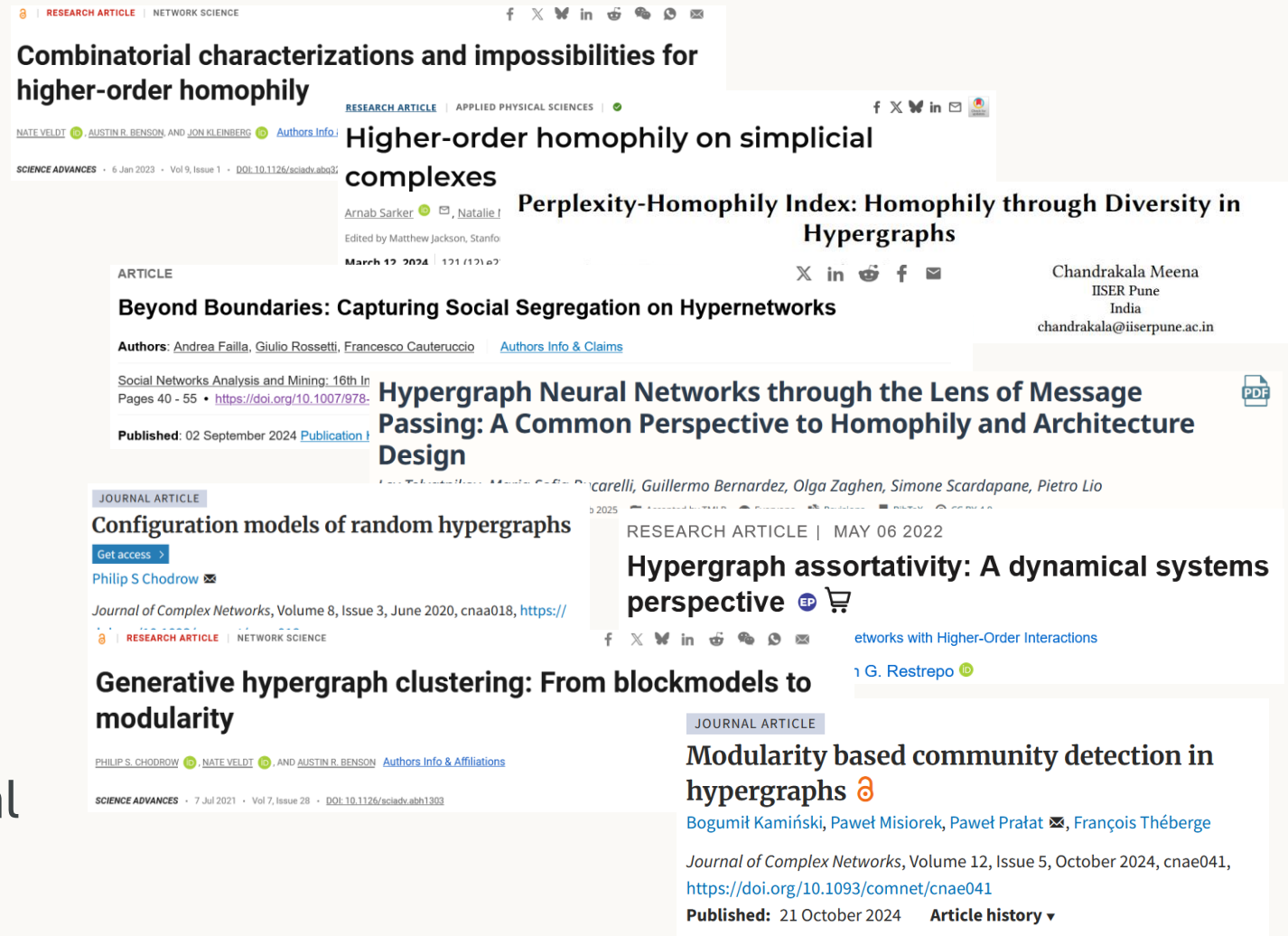
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- We provide didactic **examples**.



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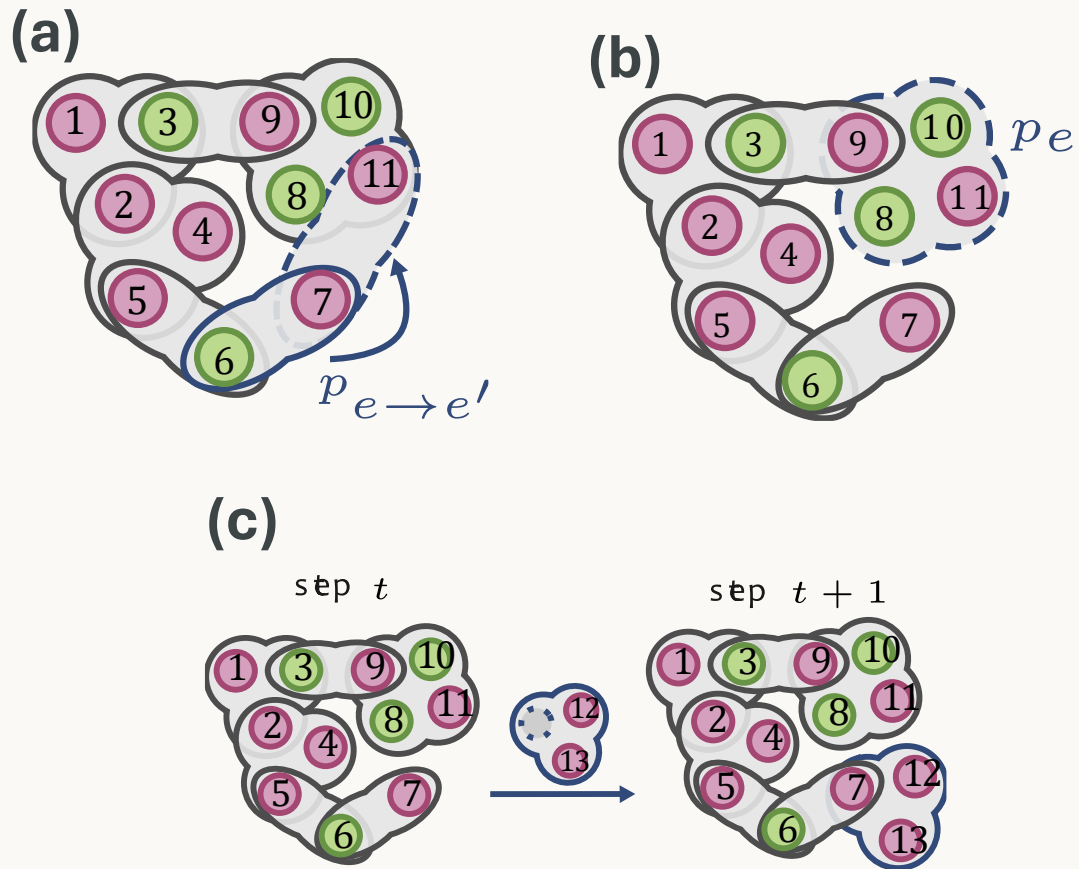
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- We provide didactic **examples**.
- We **provide guidance** on conceptual insights, limitations, challenges...



A Guide to Higher-Order Homophily

$\mathbb{P}(\mathcal{H})$

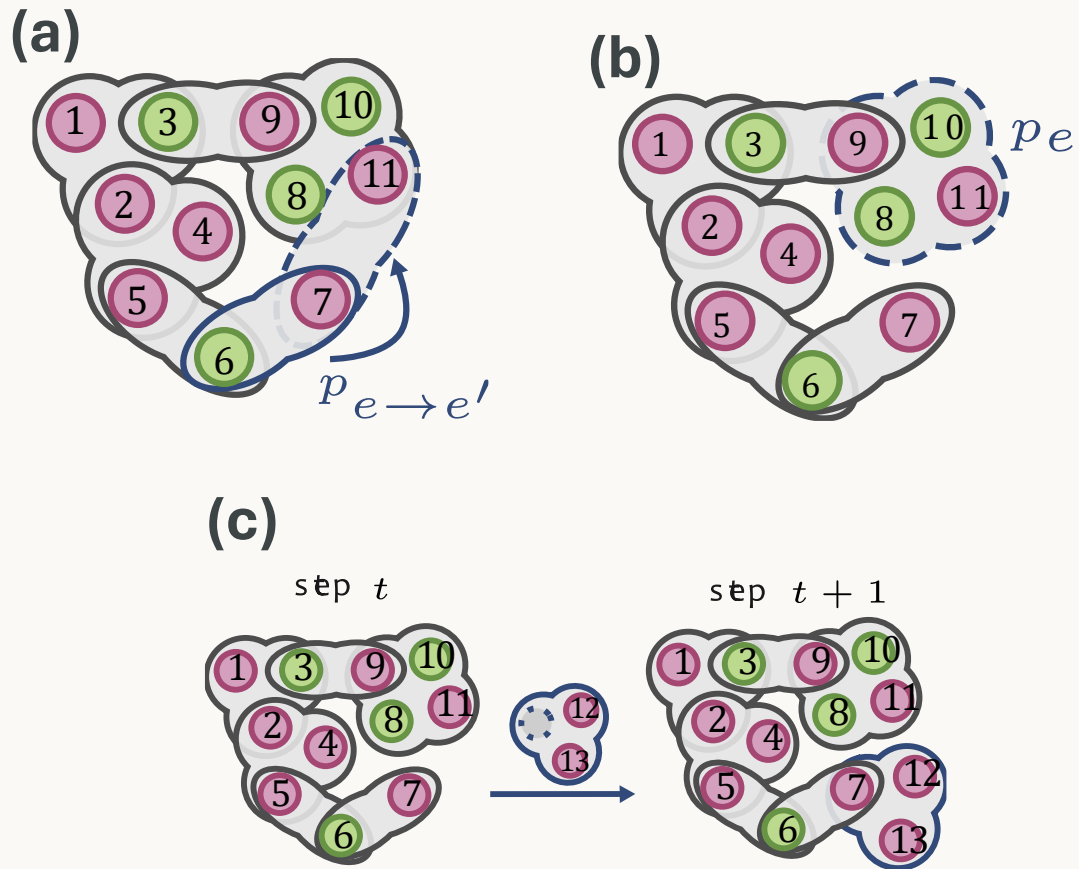


Models

- We provide an **overview of different classes** of models.
 - Null models
 - Hypergraph SBMs
 - Geometric models
 - Growing / Mechanistic models

A Guide to Higher-Order Homophily

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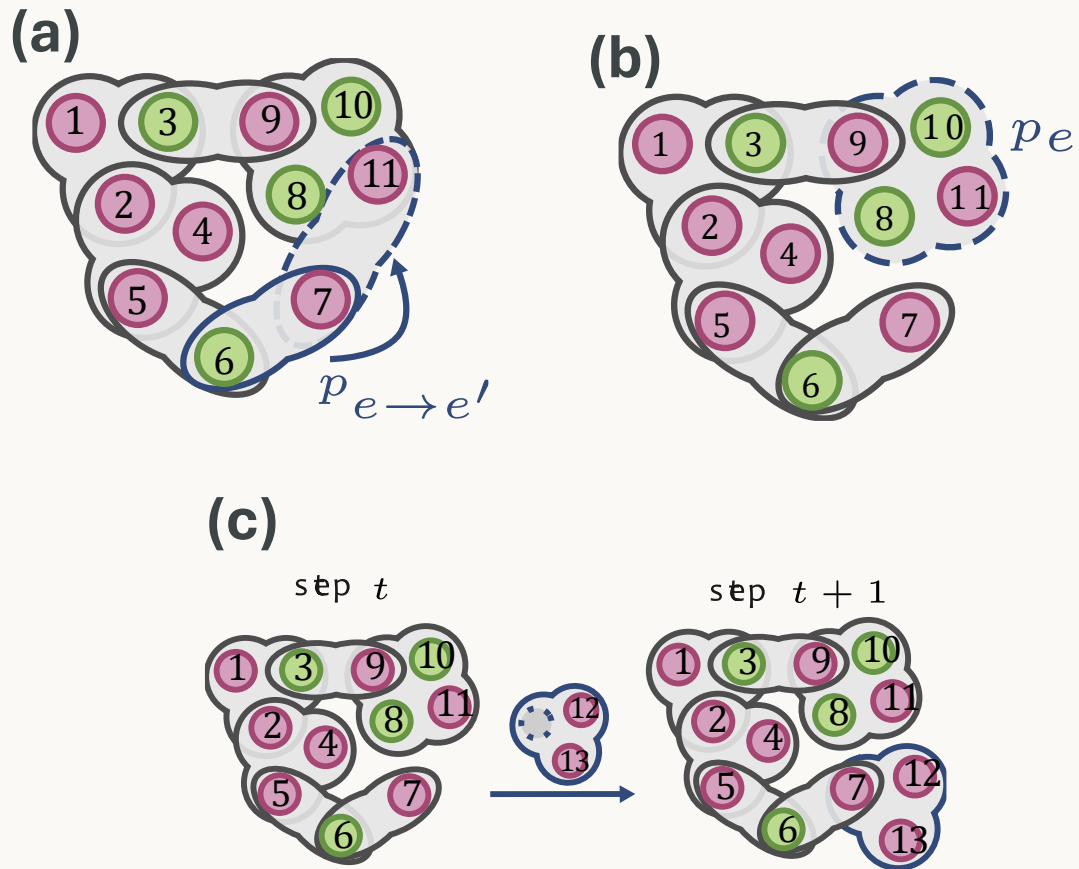


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- **Discuss their role** in for studying higher-order homophily.

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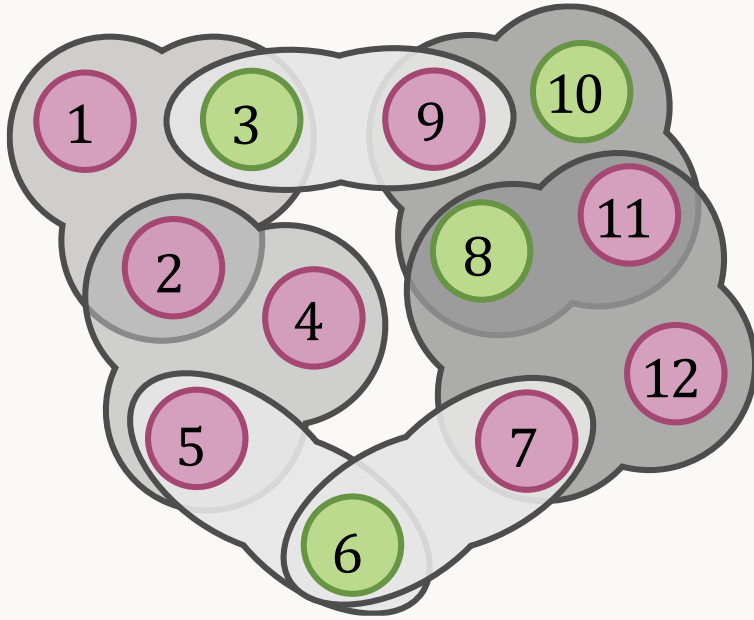
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 - Null models
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- Discuss their role in for studying higher-order homophily.
- Highlight **open questions**.

Example Measure: Hypergraph Homophily



Properties

$N = 12$ number of nodes

$c \in \{0, 1\}$ two classes

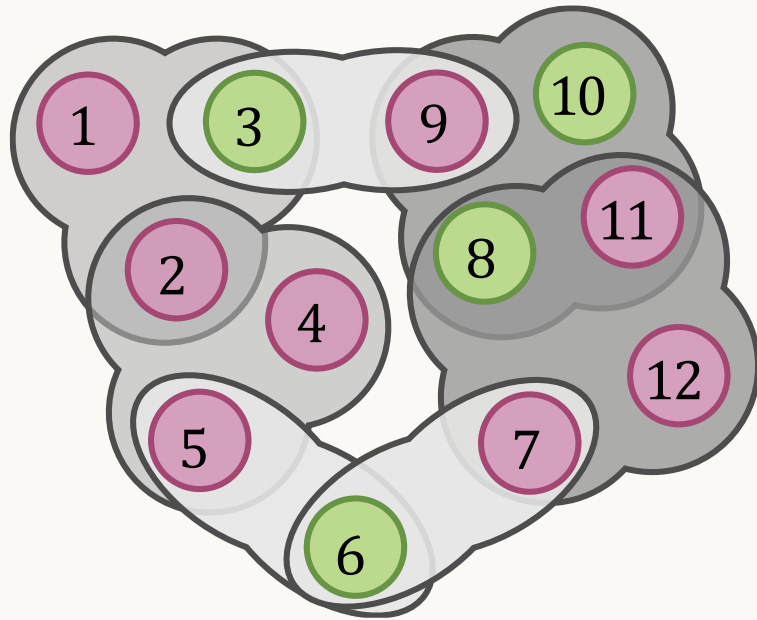
$N_0 = 8$ number of **class 0** nodes

$N_1 = 4$ number of **class 1** nodes

$E = 7$ number of hyperedges

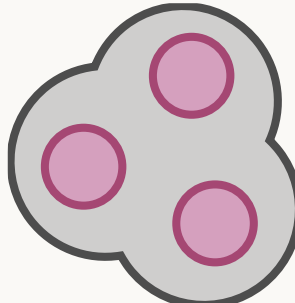
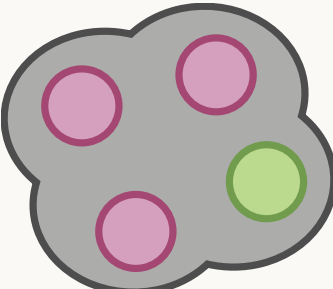
$\alpha \in \{2, 3, 4\}$ hyperedge sizes

Example Measure: Hypergraph Homophily

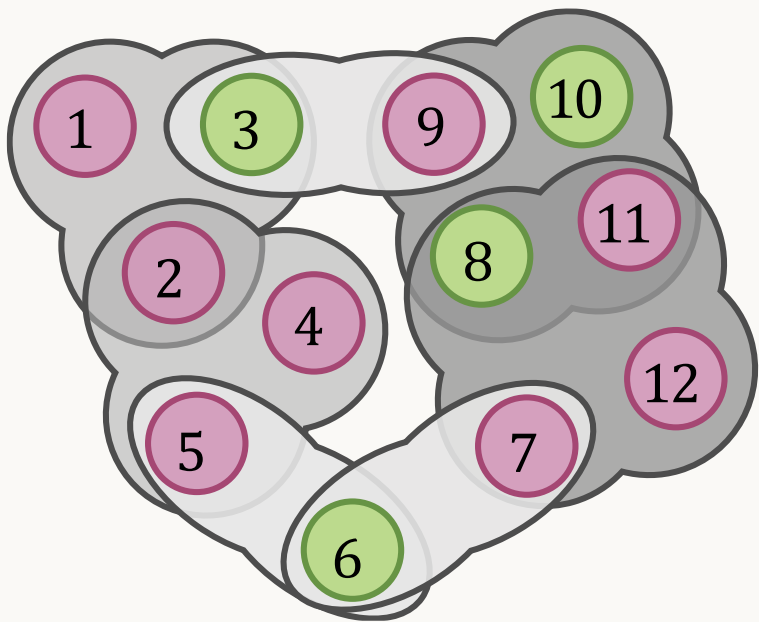


Number of hyperedges of type

$$E_{\alpha,t}^{(c)} = |\{e \in E : t_c(e) = t \wedge |e| = \alpha\}|$$

	Size	Type
	$\alpha = 2$	$t_0(e) = 1$ $t_1(e) = 1$
	$\alpha = 3$	$t_0(e) = 3$ $t_1(e) = 0$
	$\alpha = 4$	$t_0(e) = 3$ $t_1(e) = 1$

Example Measure: Hypergraph Homophily



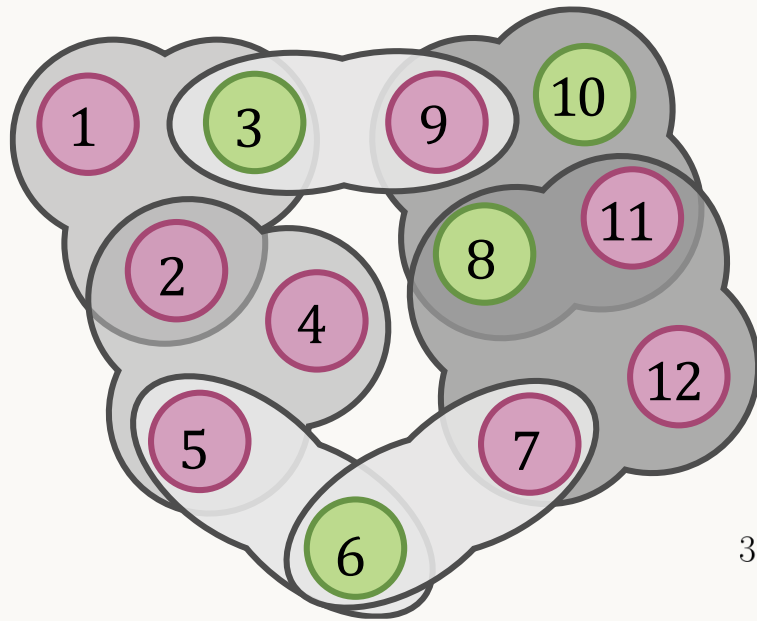
$$a_{c,\alpha,t}^{\text{VBK1}} = \frac{tE_{\alpha,t}^{(c)}}{\sum_{t'=1}^{\alpha} t'E_{\alpha,t'}^{(c)}}$$

$$b_{c,\alpha,t}^{\text{VBK1}} = \frac{\binom{N_c-1}{t-1}\binom{N-N_c}{\alpha-t}}{\binom{N-1}{\alpha-1}}$$

$$h_{c,\alpha,t}^{\text{VBK1}} = \frac{a_{c,\alpha,t}^{\text{VBK1}}}{b_{c,\alpha,t}^{\text{VBK1}}}$$

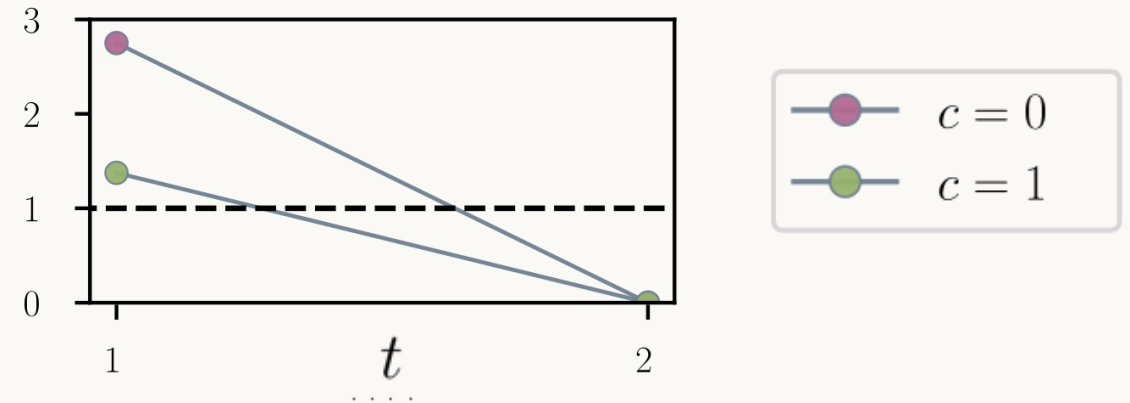
c	α	t	$E_{\alpha,t}^{(c)}$	$a_{c,\alpha,t}^{\text{VBK1}}$	$b_{c,\alpha,t}^{\text{VBK1}}$	$h_{c,\alpha,t}^{\text{VBK1}}$
0	2	0	0	—	—	—
		1	3	3/3	4/11	11/4
		2	0	0/3	7/11	0
	3	0	0	—	—	—
		1	0	0/5	6/55	0
		2	1	2/5	28/55	11/14
		3	1	3/5	21/55	22/14
	4	0	0	—	—	—
		1	0	0/5	4/165	0
		2	1	2/5	42/165	33/21
		3	1	3/5	85/165	99/85
		4	0	0/5	35/165	0
1	2	0	0	—	—	—
		1	3	3/3	8/11	11/8
		2	0	0/3	3/11	0
	3	0	0	—	—	—
		1	1	1/1	28/55	55/28
		2	0	0/1	24/55	0
		3	0	0/1	3/55	0
	4	0	0	—	—	—
		1	1	1/3	56/165	55/56
		2	1	2/3	84/165	55/42
		3	0	0/3	24/165	0
		4	0	0/3	1/165	0

Example Measure: Hypergraph Homophily



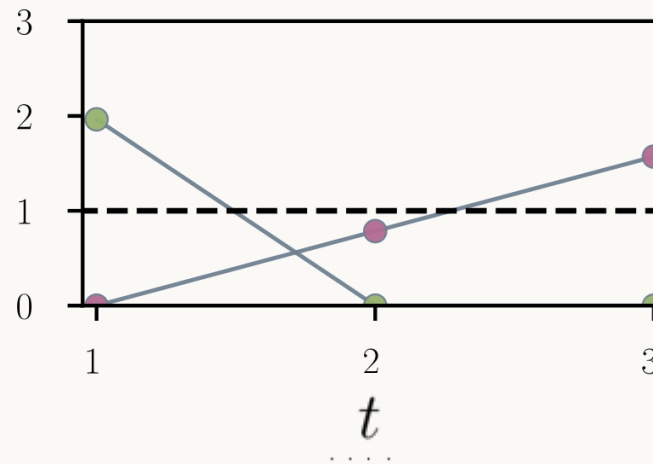
$\alpha = 2$

$h_{c,2,t}^{\text{VBK1}}$



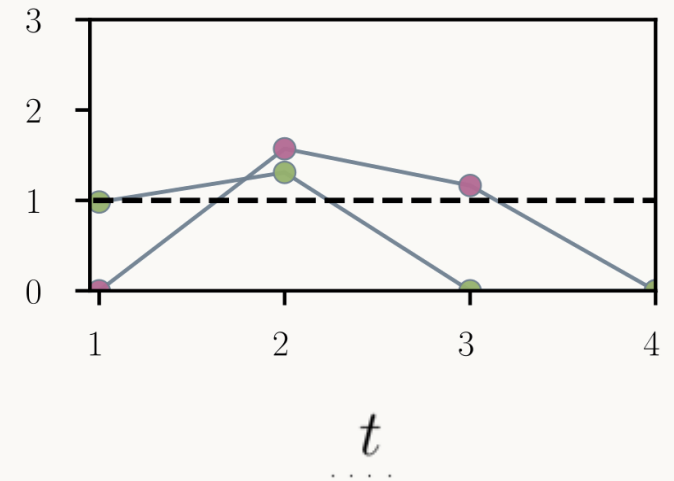
$\alpha = 3$

$h_{c,3,t}^{\text{VBK1}}$



$\alpha = 4$

$h_{c,4,t}^{\text{VBK1}}$



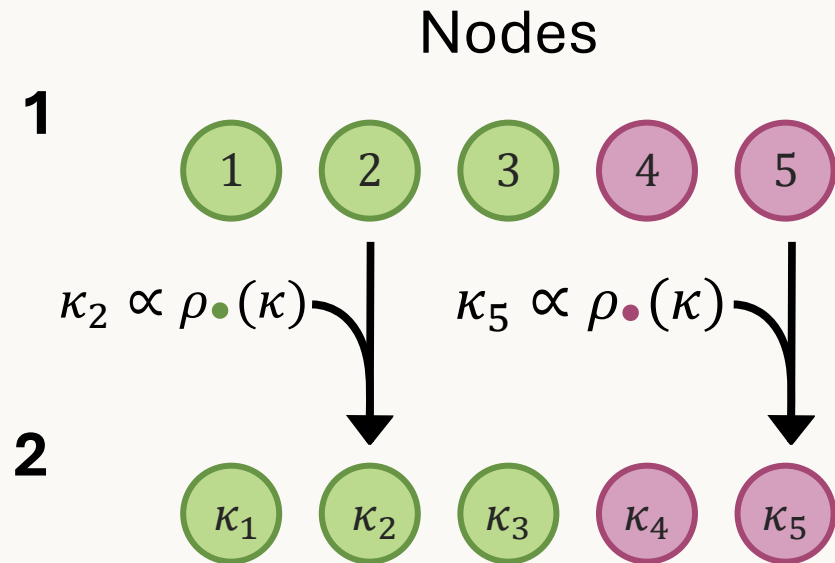
Example Model: H3 Model

Nodes

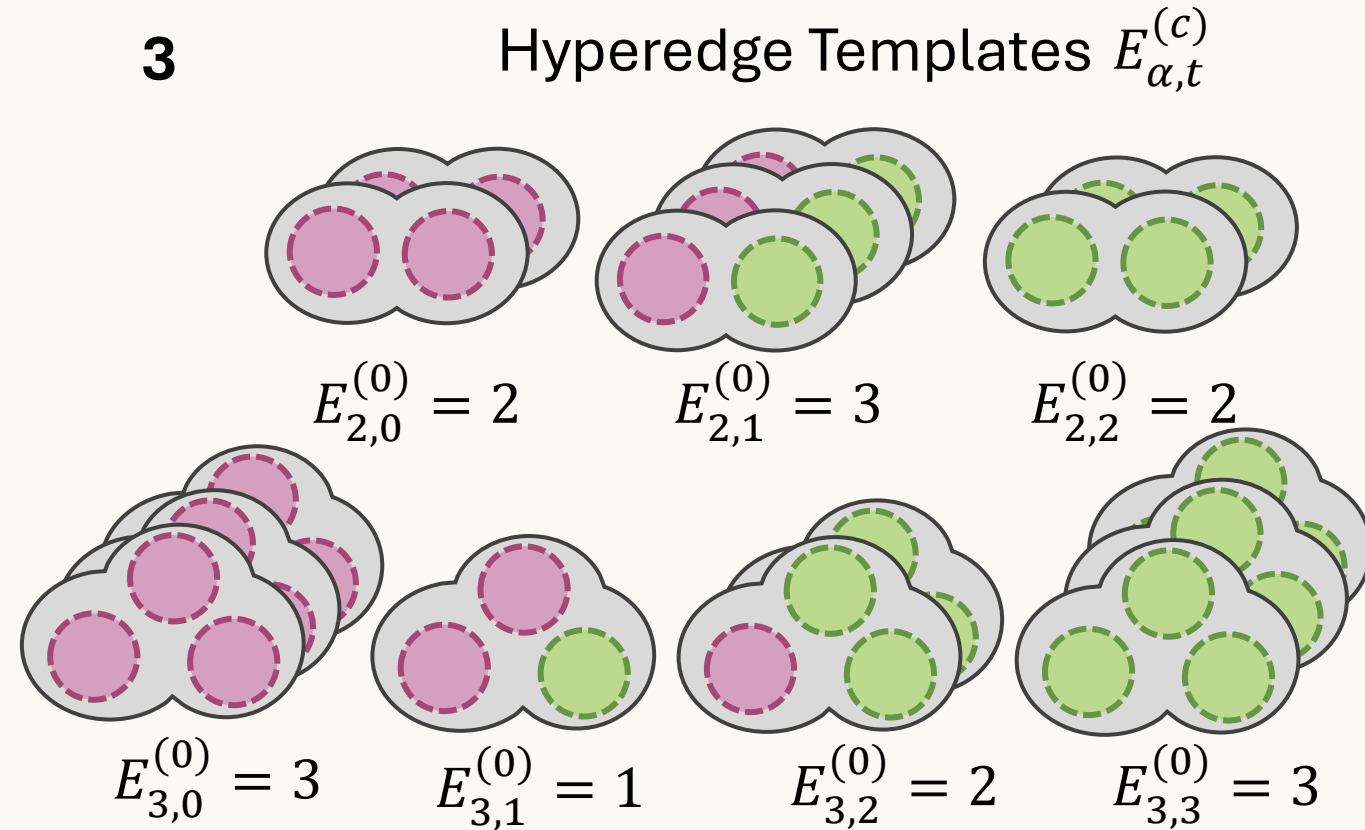
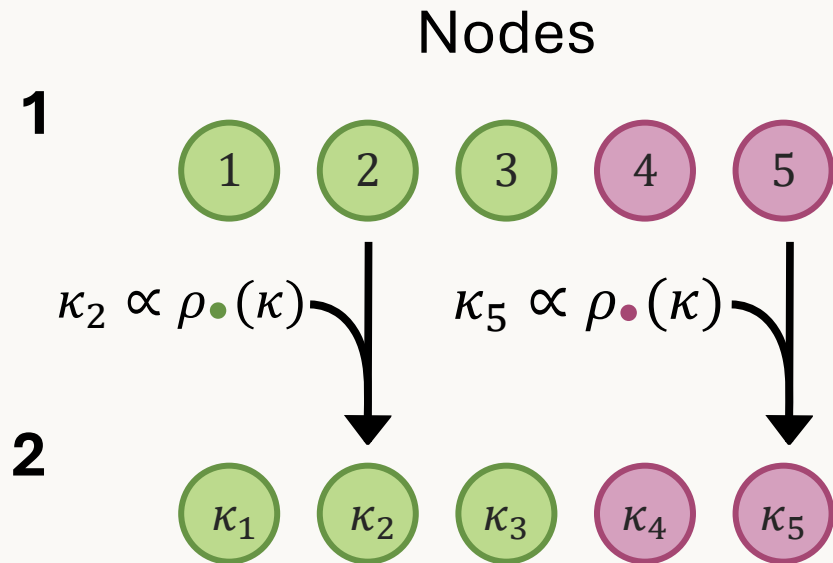
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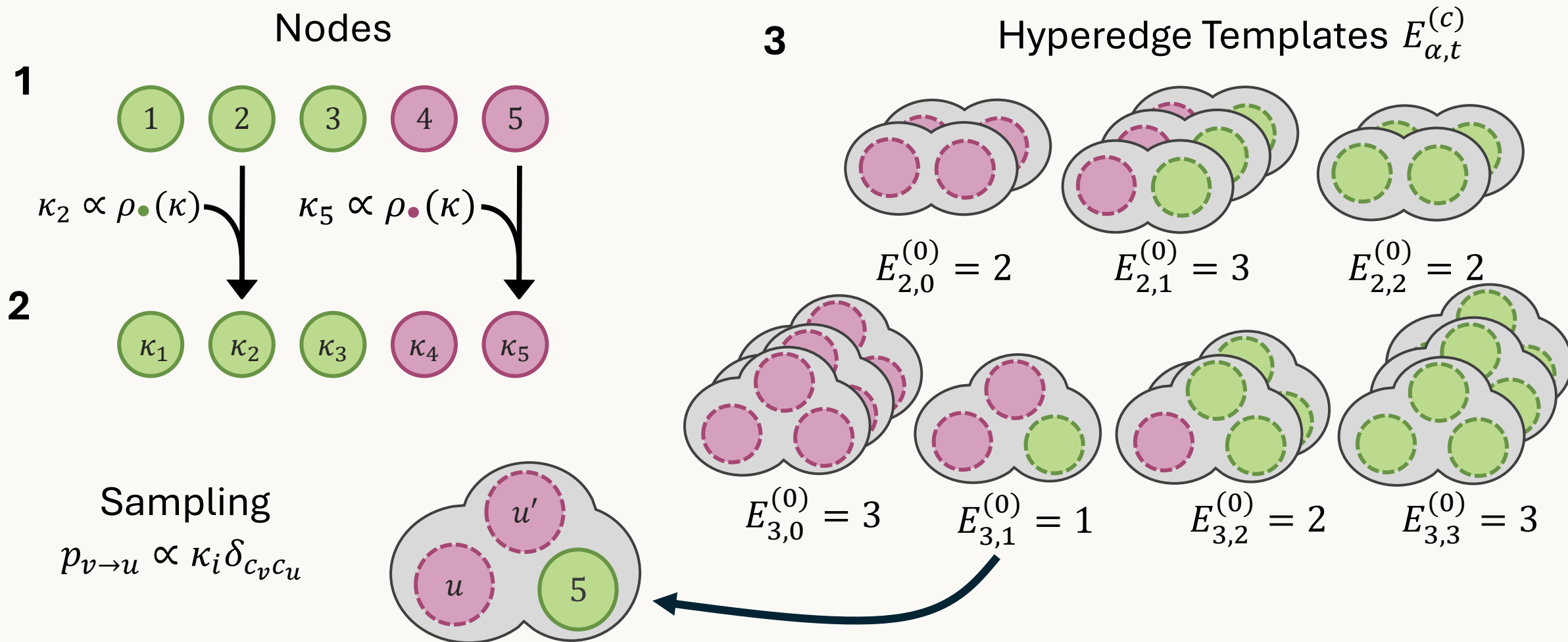
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Laber, Dies, Ehlert et al. Comm. Phys. **9** 13 (2026)

Conclusion

Key Insights

- Hyperedges are **not either within- or between-class**. Higher-order mixing patterns are much richer.
- Mixing patterns can **vary with hyperedge size**.
- There is a **lot of methods** out there, but less effort to re-connect theory in the social sciences and a lack of synthesis.

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- How should we **summarize the high-dimensional information** that attributed hypergraphs provide?
- How do we create hypergraph **models that are principled and computationally efficient**?
- **Do higher-order mixing patterns matter** beyond pairwise affinity?

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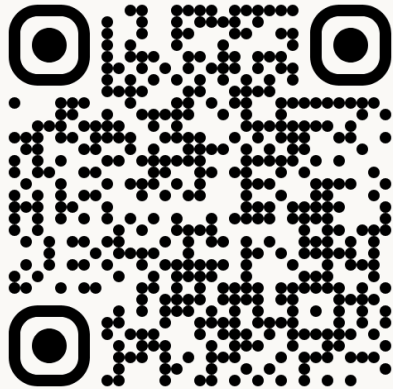
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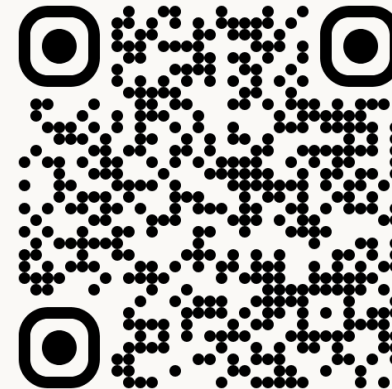
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<https://arxiv.org/abs/2606.02537v1>



Effects of higher-order interactions and homophily on information access inequality

<https://doi.org/10.1038/s42005-025-02445-y>



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