Mixing Patterns in Higher-Order Networks: A Review

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Homophily & Heterophily

Homophily is the above-chance occurrence of interactions between similar nodes.

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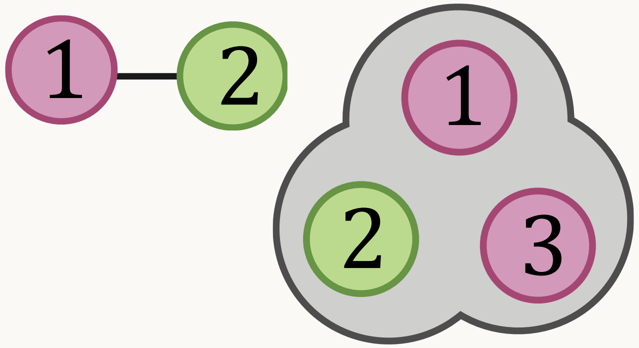
Connectivity

(Hyper)edges indicating, e.g., co-authorship, discussions, teams, …



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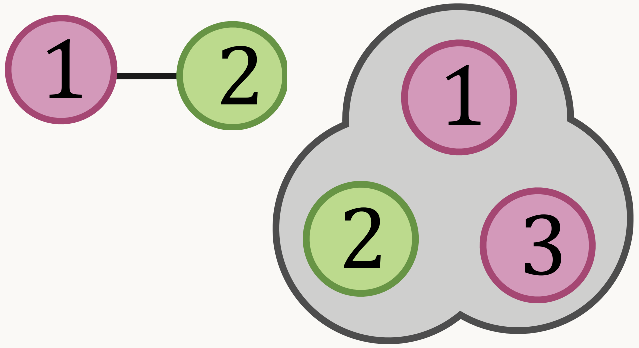
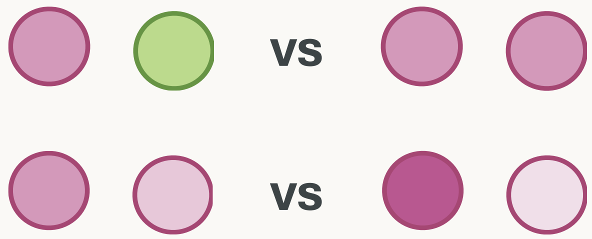


Similarity

- *discrete attributes*: equality
- *continuous attributes*: inner products and distances



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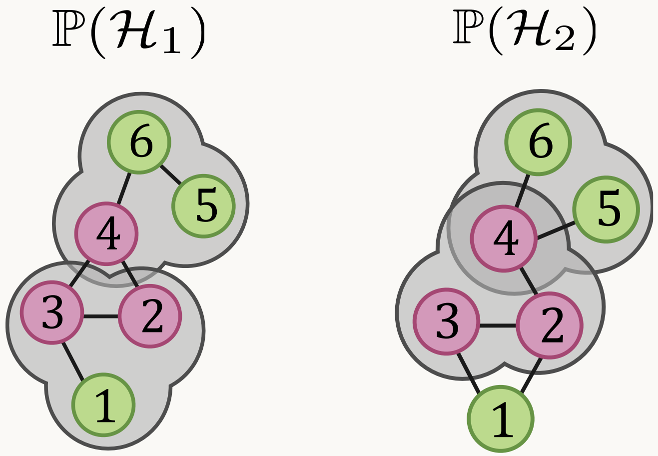
Null model

preserve relative class sizes, and structural properties, e.g., average degree.



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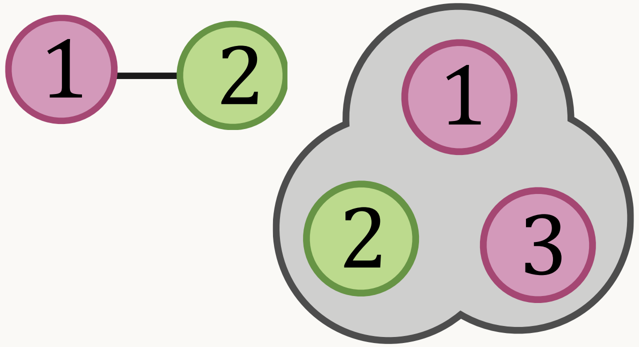

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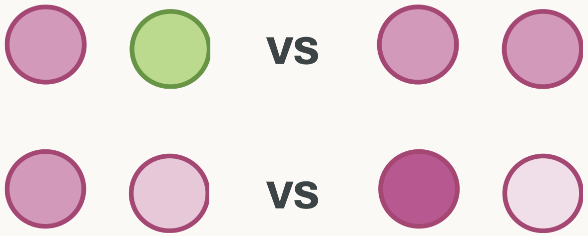

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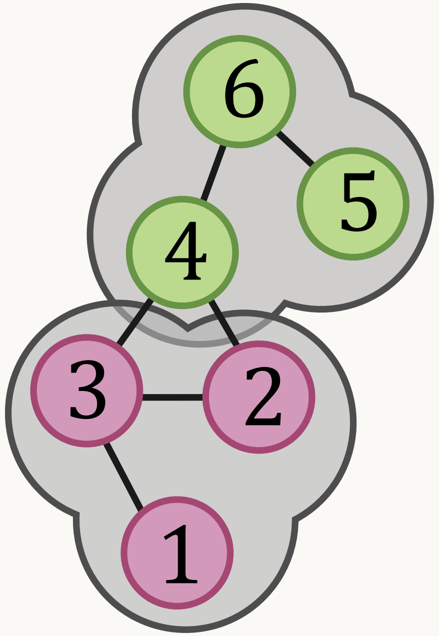

homophily

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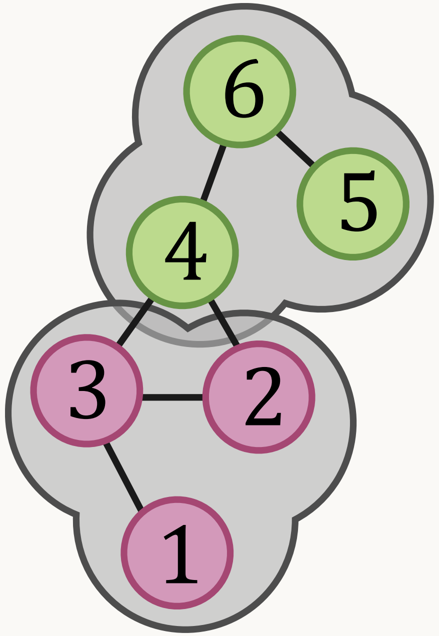

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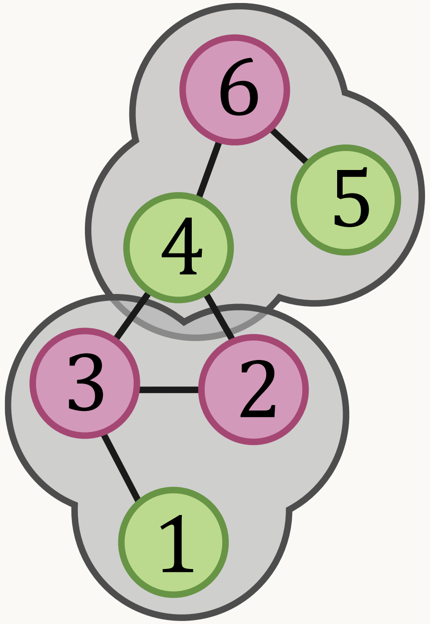
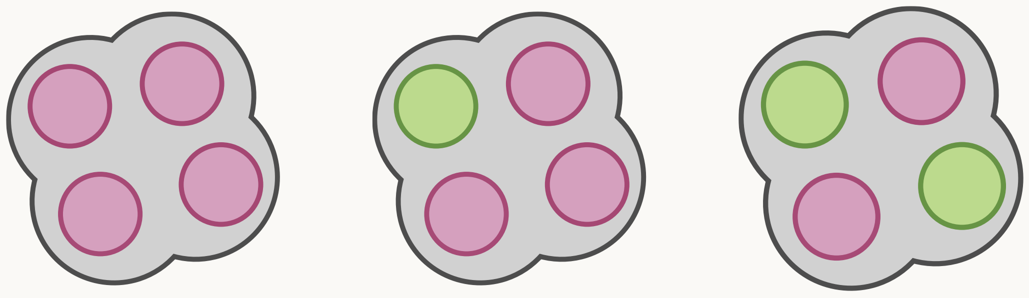


heterophily

Higher-Order Homophily

Do **within-class** hyperedges make sense?



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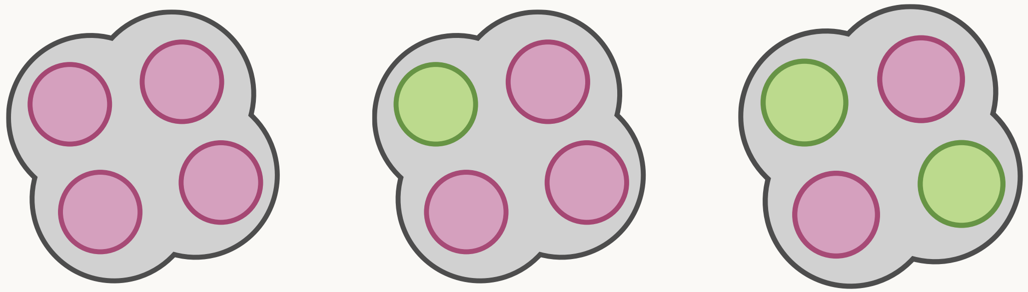
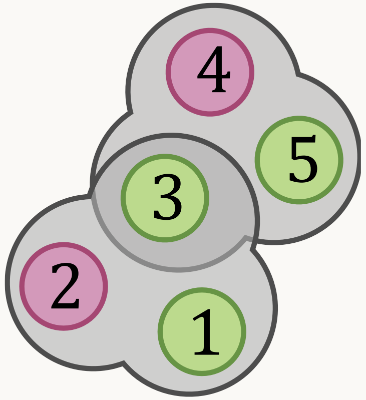


There are combinatorial **constraints**.



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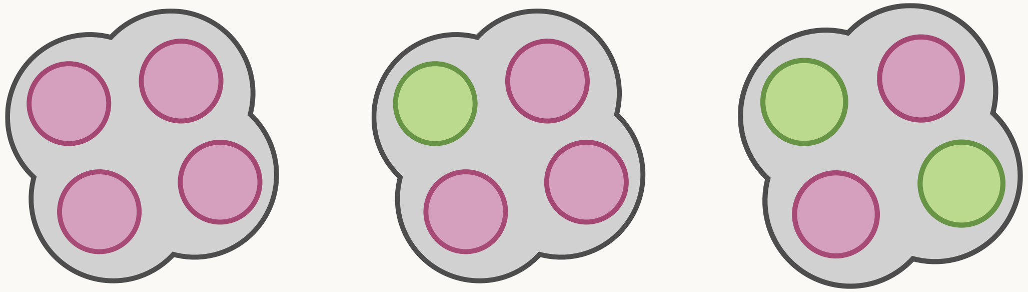

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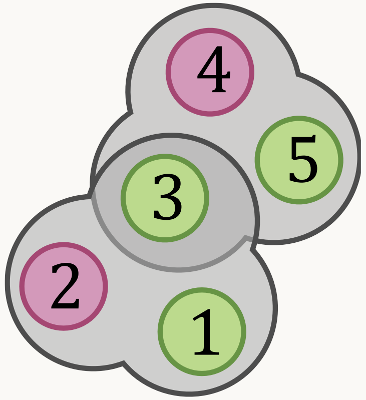
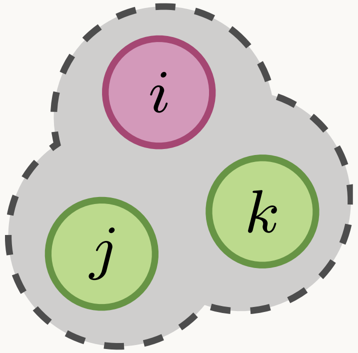


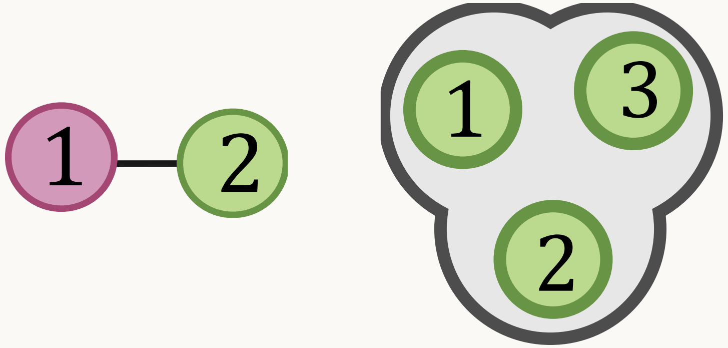
There are **a lot** of possible hyperedges.

$$E^{\max}_{n,\alpha} = \binom{n}{\alpha}$$

$$\frac{E^{\max}_{1000,4}}{E^{\max}_{1000,2}} \approx 8 \cdot 10^4$$



Mixing patterns can **vary by hyperedge size.**



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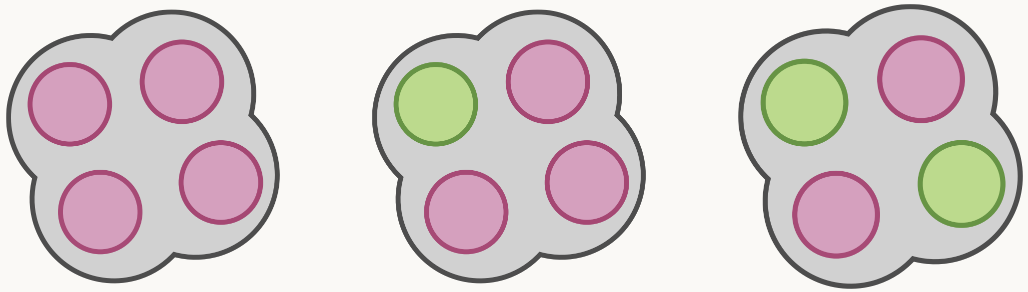

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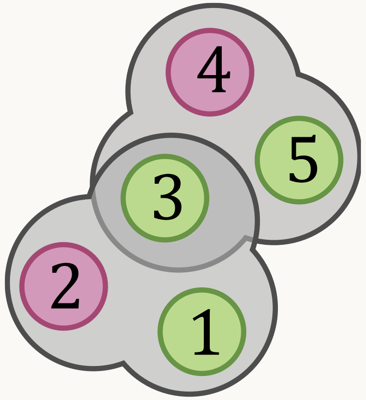

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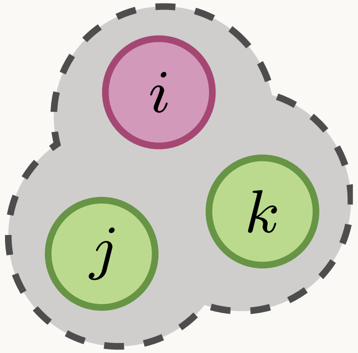

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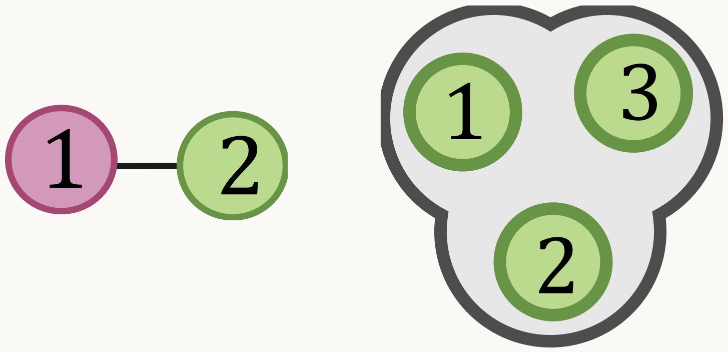

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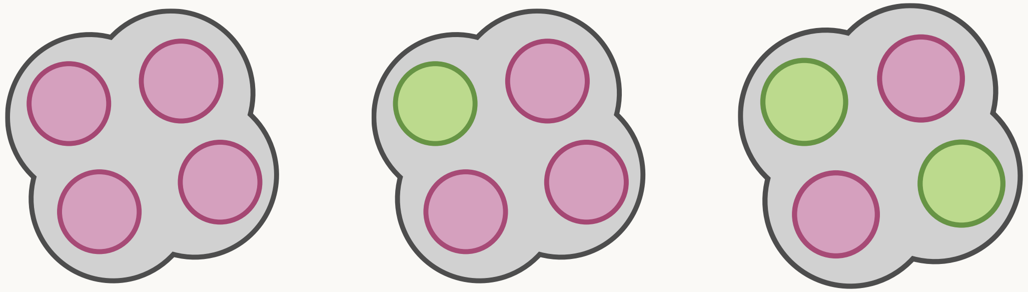

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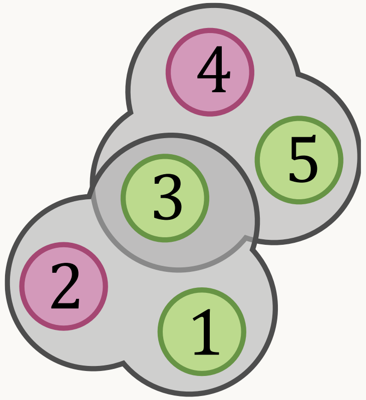

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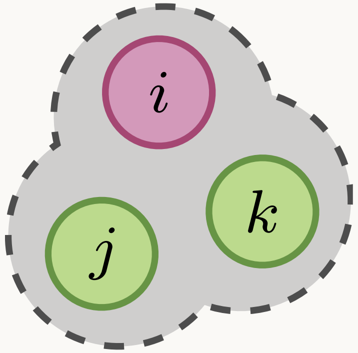

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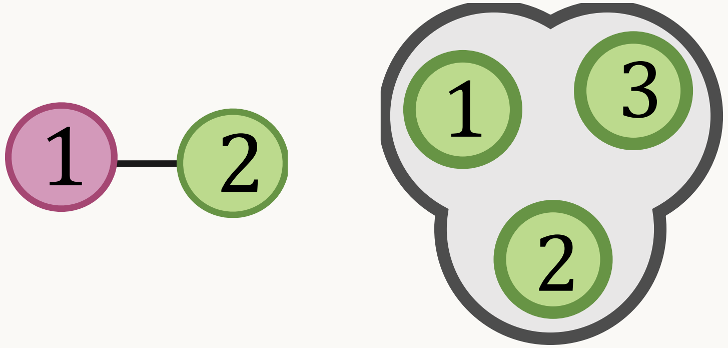


A Guide to Higher-Order Homophily

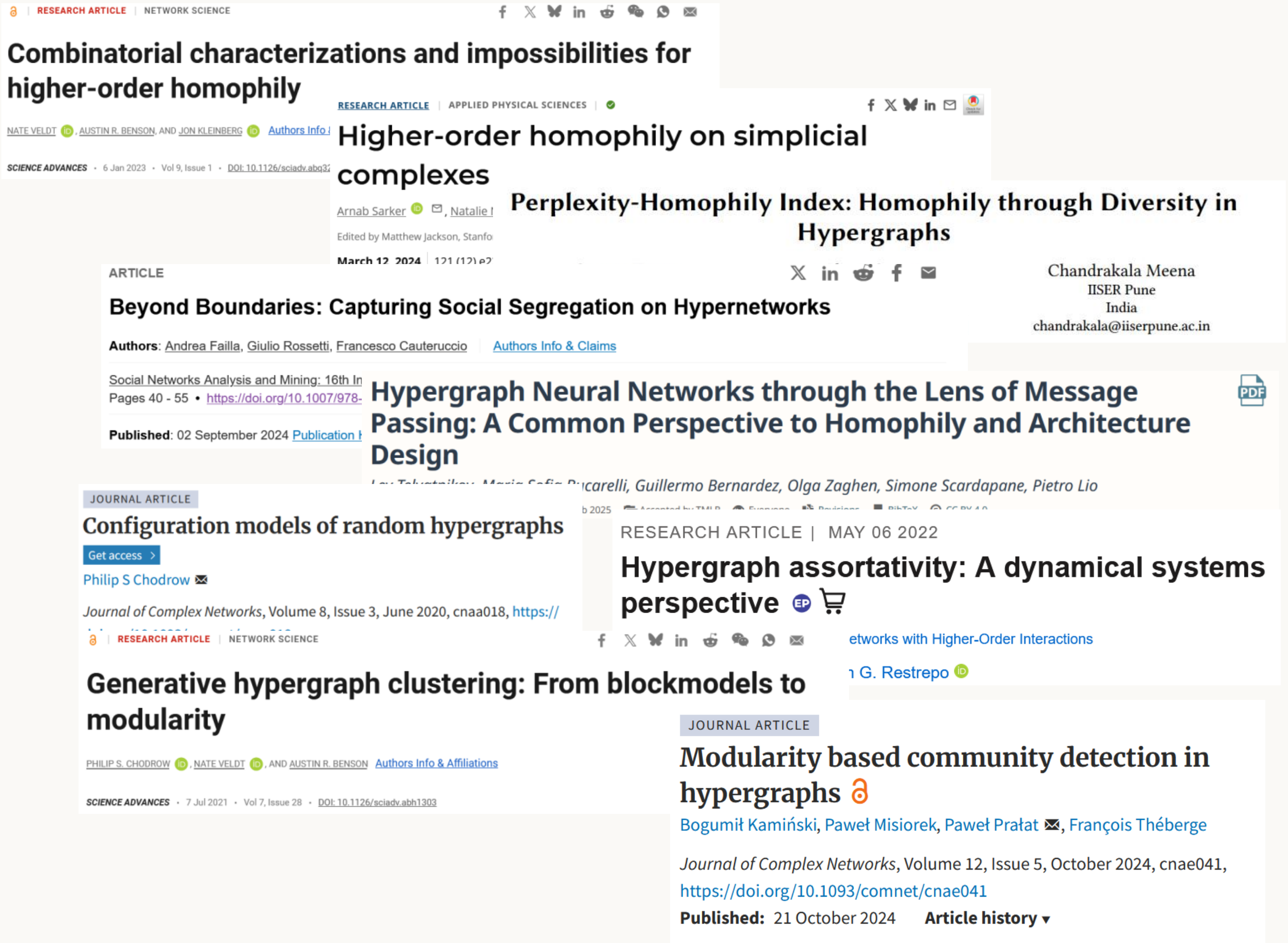
Measures

Models

A Guide to Higher-Order Homophily

Measures

- We survey different **definitions**.
  - Hypergraph homophily
  - Simplicial homophily
  - Perplexity-Homophily index
  - Random Walk HyperSegregation
  - Message Passing Homophily
  - Hypergraph assortativity
  - Hypergraph Modularity
  - Clique Projection



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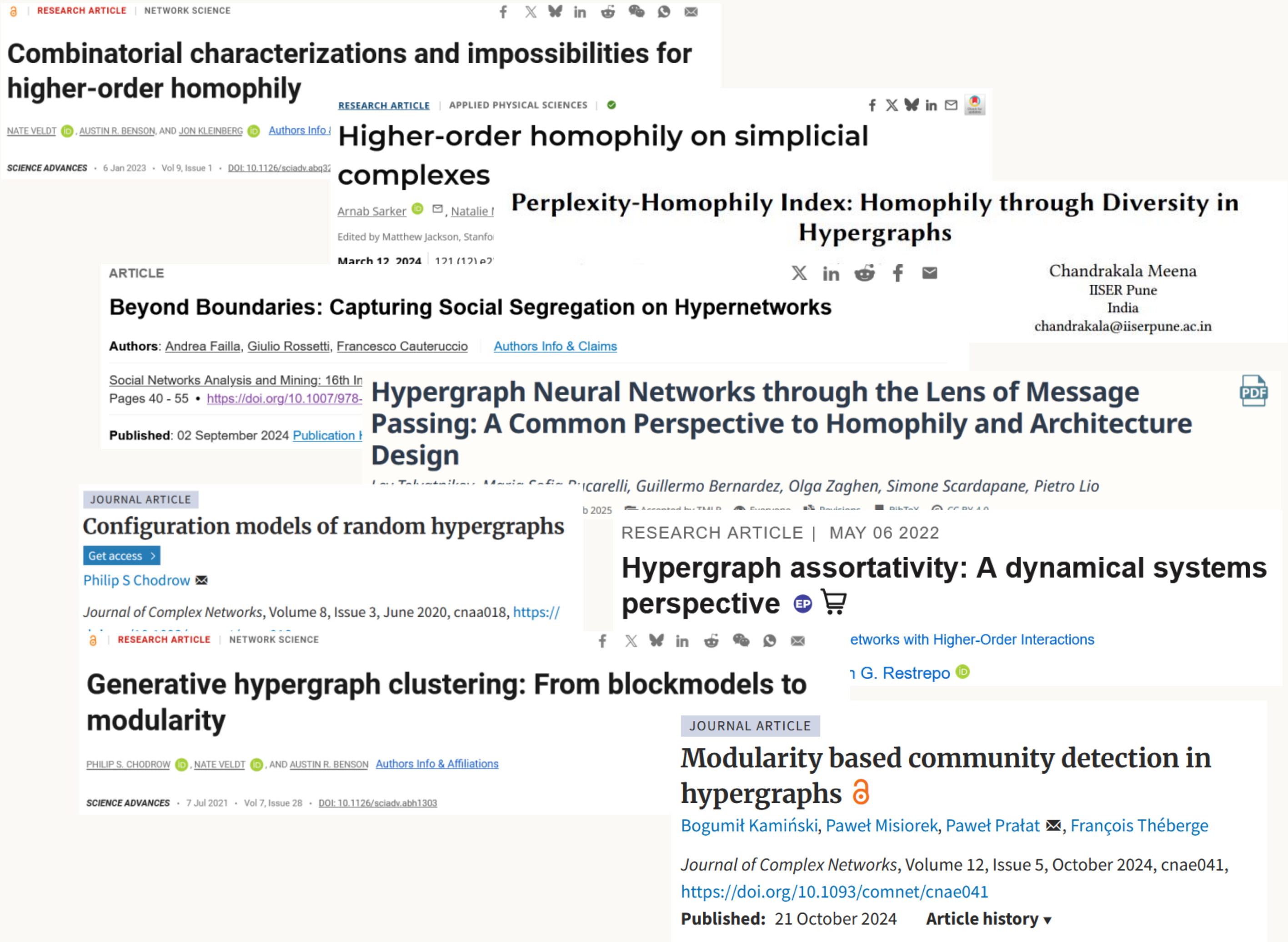

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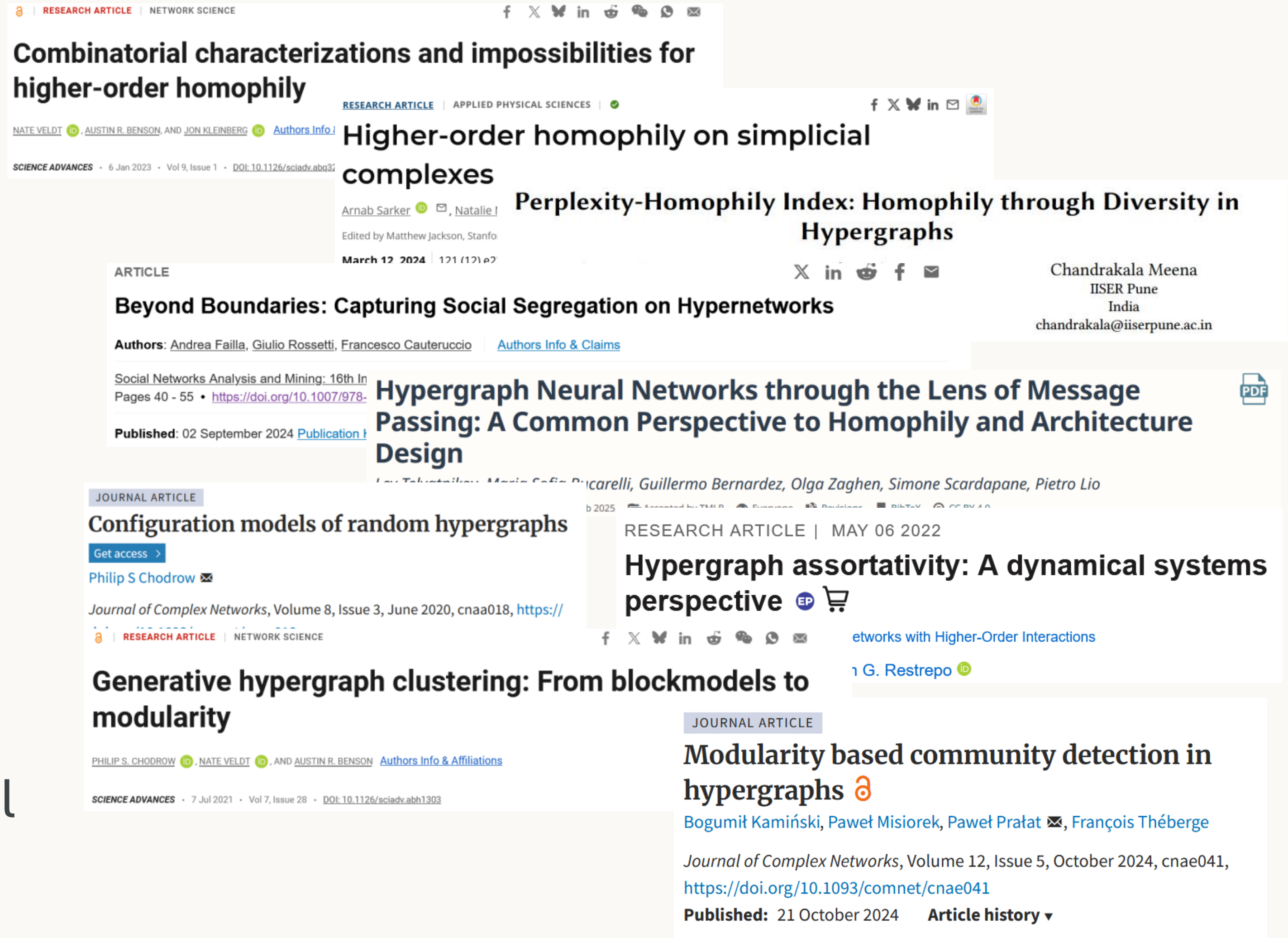
- We provide didactic **examples**.
- We **provide guidance** on conceptual insights, limitations, challenges…



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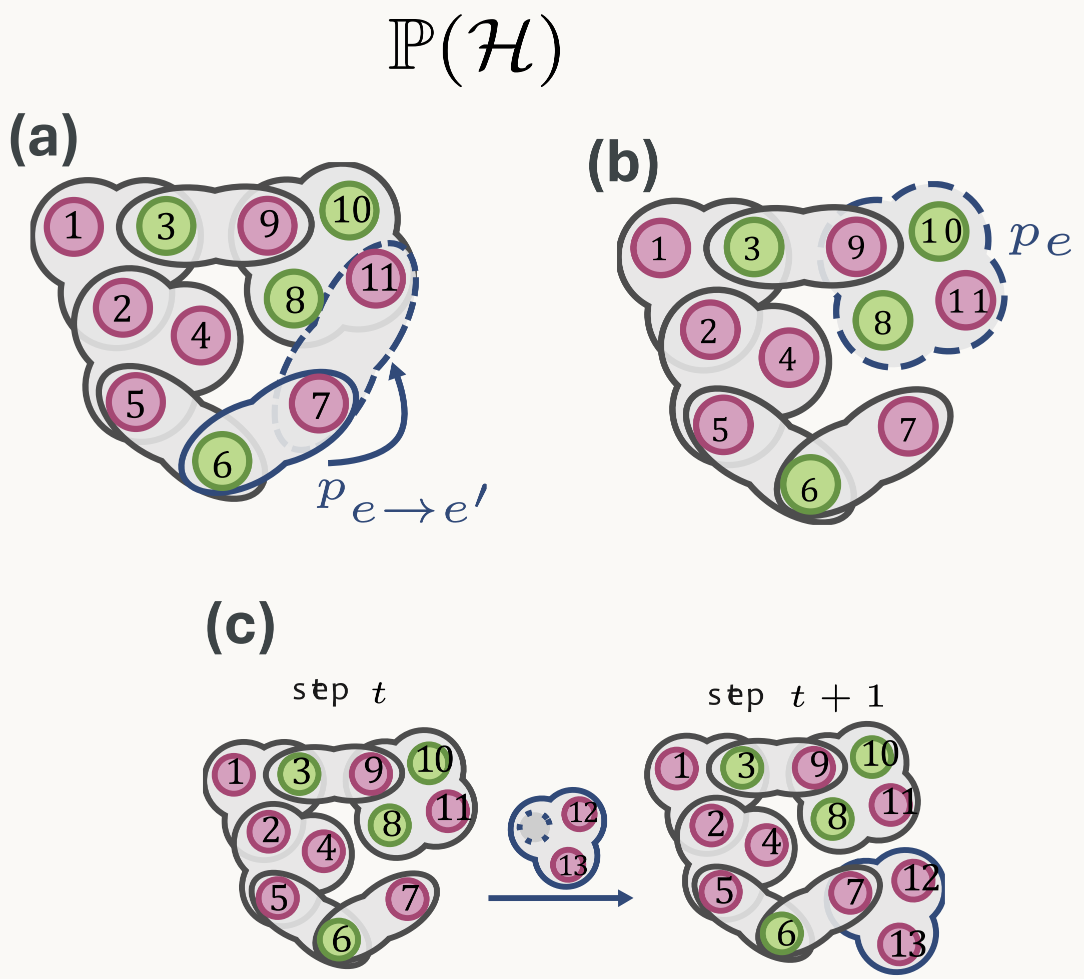

Models

- We provide an **overview of different classes** of models.
  - Null models
  - Hypergraph SBMs
  - Geometric models
  - Growing / Mechanistic models

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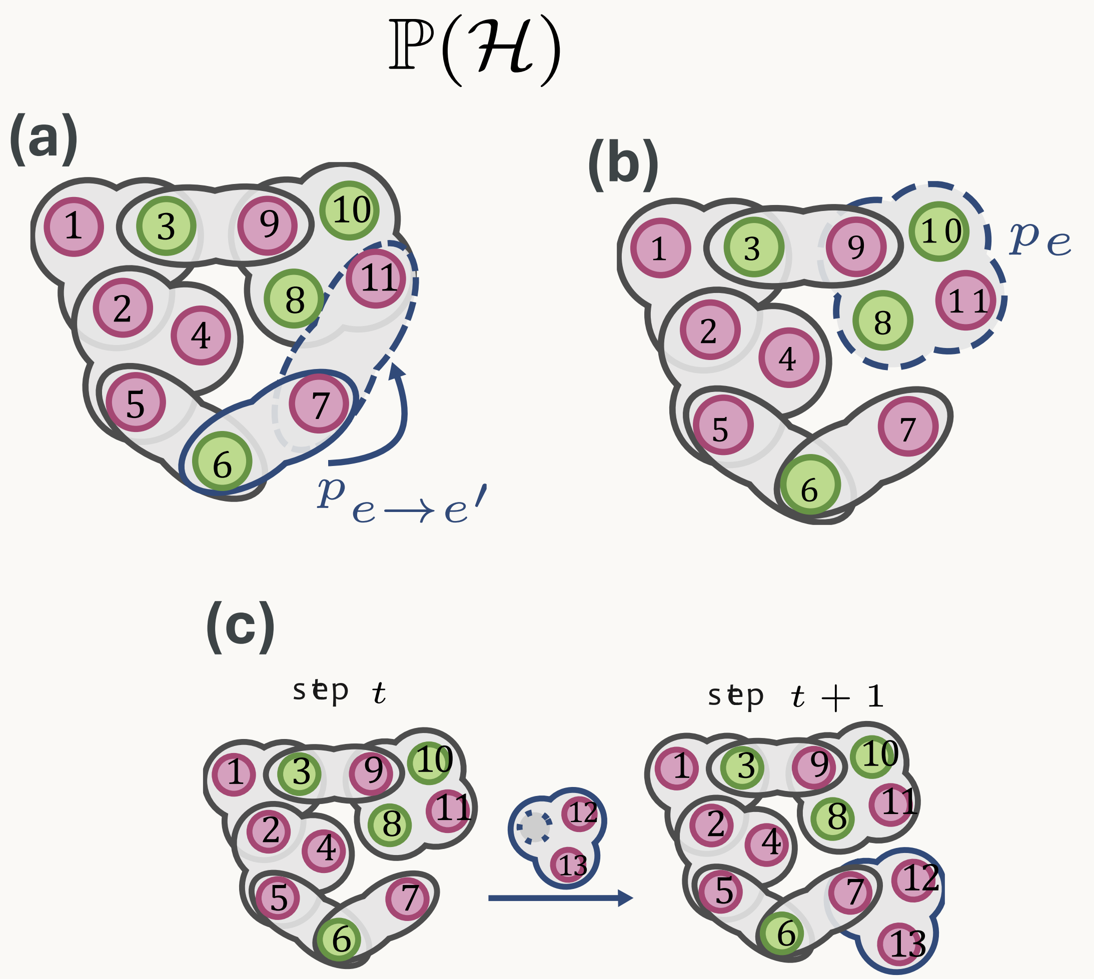

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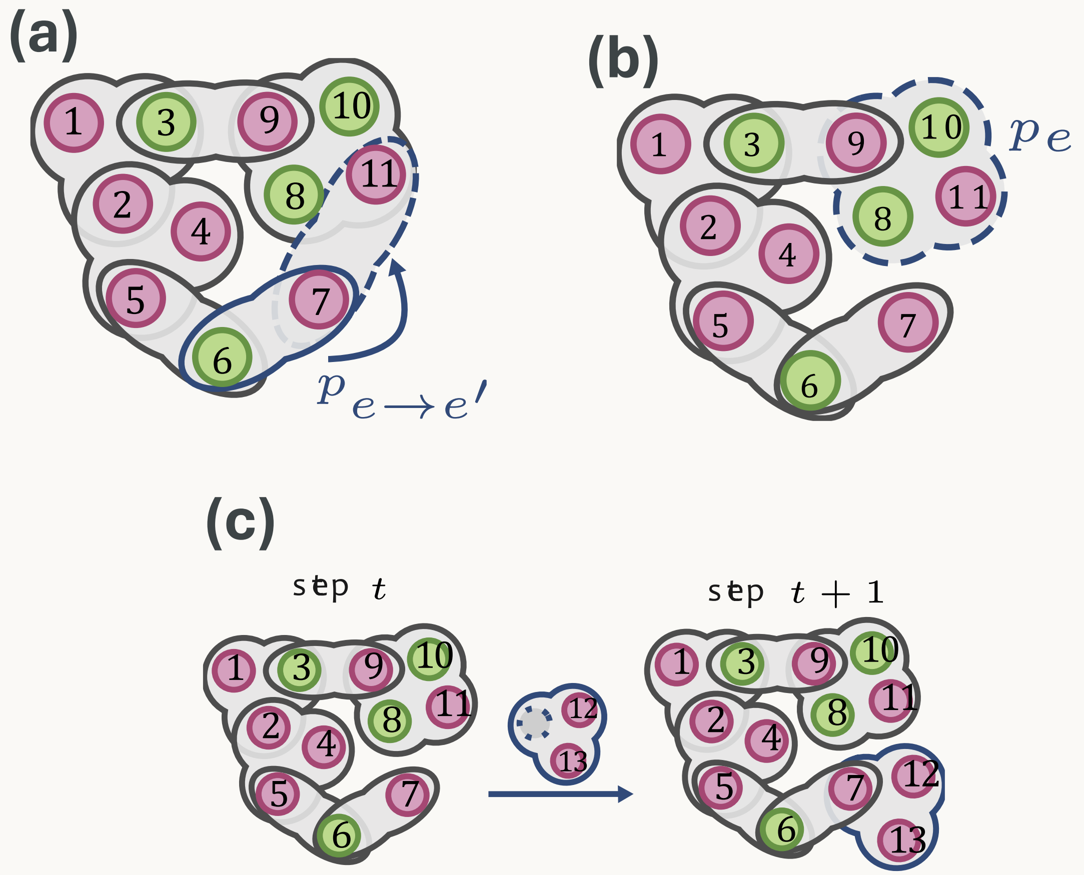
- **Discuss their role** in for studying higher-order homophily.

A Guide to Higher-Order Homophily

$\mathbb{P}(\mathcal{H})$



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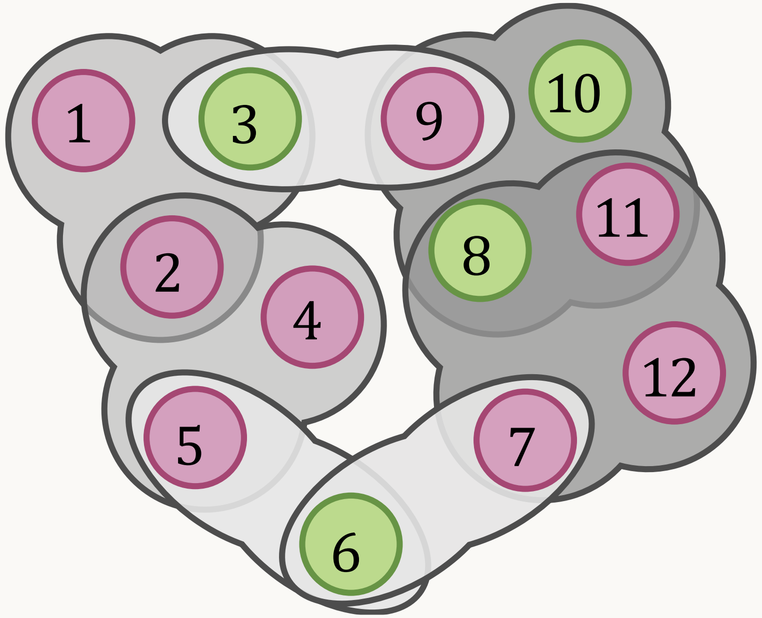
- Discuss their role in for studying higher-order homophily.
- Highlight **open questions**.

Example Measure: Hypergraph Homophily



Properties

$N = 12$ number of nodes

$c \in \{0, 1\}$ two classes

$N_0 = 8$ number of class 0 nodes

$N_1 = 4$ number of class 1 nodes

$E = 7$ number of hyperedges

$\alpha \in \{2,3,4\}$ hyperedge sizes

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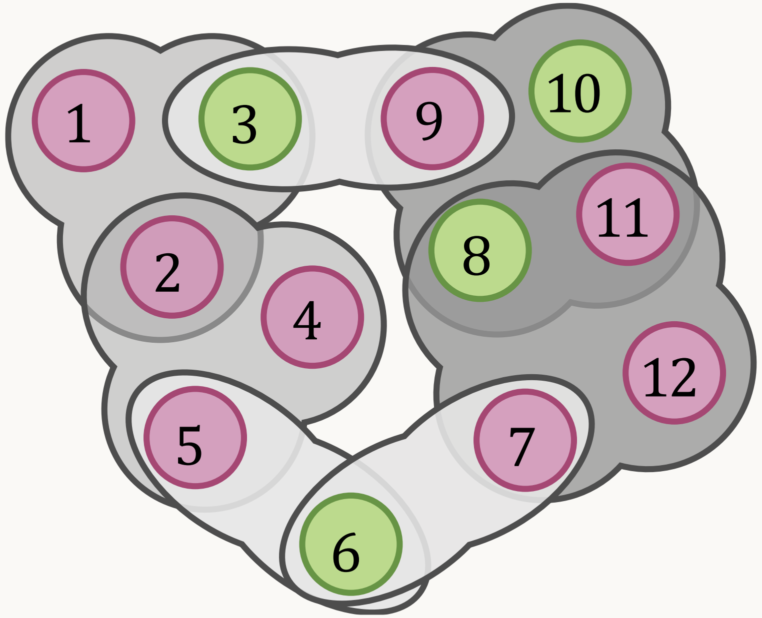
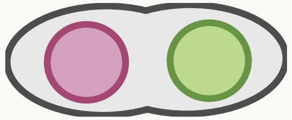
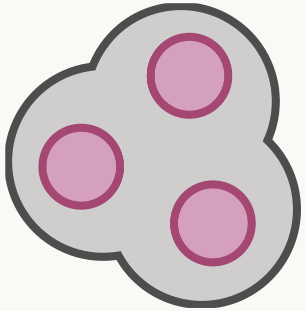
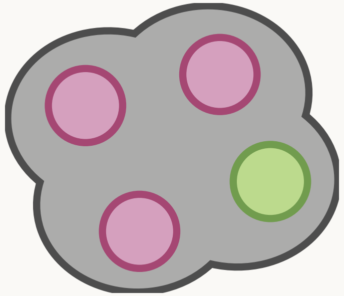
Example Measure: Hypergraph Homophily



Number of hyperedges of type

$$E_{\alpha,t}^{(c)} = |\{e \in E: t_c(e) = t \wedge |e| = \alpha\}|$$

	Size	Type
	$\alpha = 2$	$t_0(e) = 1$ $t_1(e) = 1$
	$\alpha = 3$	$t_0(e) = 3$ $t_1(e) = 0$
	$\alpha = 4$	$t_0(e) = 3$ $t_1(e) = 1$

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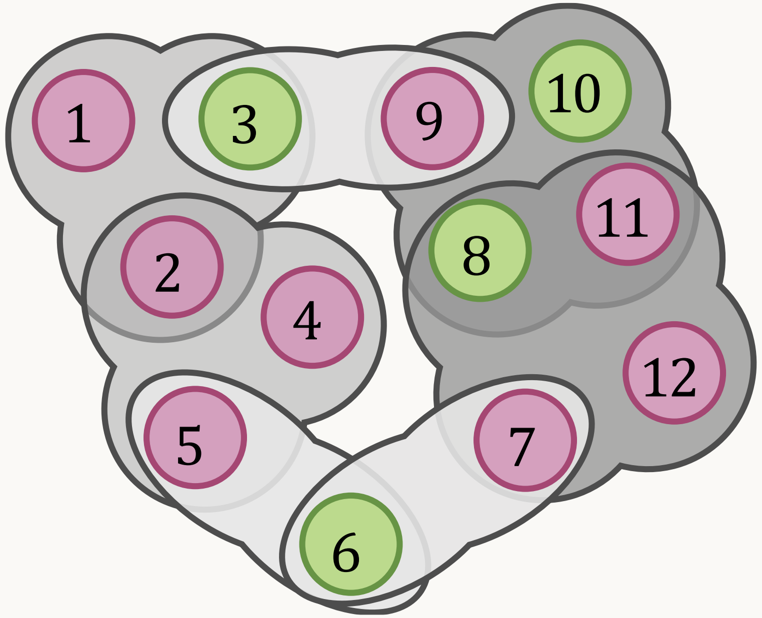

$$a^{\mathrm{VBK1}}_{c,\alpha,t} = \frac{tE^{(c)}_{\alpha,t}}{\sum_{t'=1}^{\alpha} t' E^{(c)}_{\alpha,t'}}$$

$$b^{\mathrm{VBK1}}_{c,\alpha,t} = \frac{\binom{N_c-1}{t-1}\binom{N-N_c}{\alpha-t}}{\binom{N-1}{\alpha-1}}$$

$$h^{\mathrm{VBK1}}_{c,\alpha,t} = \frac{a^{\mathrm{VBK1}}_{c,\alpha,t}}{b^{\mathrm{VBK1}}_{c,\alpha,t}}$$

c	α	t	$E^{(c)}_{\alpha,t}$	$a^{\mathrm{VBK1}}_{c,\alpha,t}$	$b^{\mathrm{VBK1}}_{c,\alpha,t}$	$h^{\mathrm{VBK1}}_{c,\alpha,t}$
0	2	0	0	—	—	—
		1	3	3/3	4/11	11/4
		2	0	0/3	7/11	0
	3	0	0	—	—	—
		1	0	0/5	6/55	0
		2	1	2/5	28/55	11/14
		3	1	3/5	21/55	22/14
	4	0	0	—	—	—
		1	0	0/5	4/165	0
		2	1	2/5	42/165	33/21
		3	1	3/5	85/165	99/85
		4	0	0/5	35/165	0
1	2	0	0	—	—	—
		1	3	3/3	8/11	11/8
		2	0	0/3	3/11	0
	3	0	0	—	—	—
		1	1	1/1	28/55	55/28
		2	0	0/1	24/55	0
		3	0	0/1	3/55	0
	4	0	0	—	—	—
		1	1	1/3	56/165	55/56
		2	1	2/3	84/165	55/42
		3	0	0/3	24/165	0
		4	0	0/3	1/165	0

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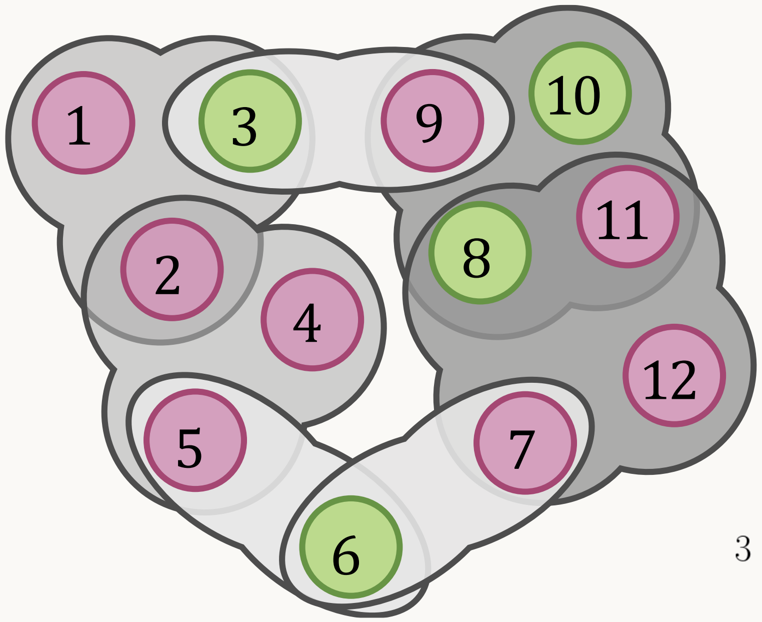

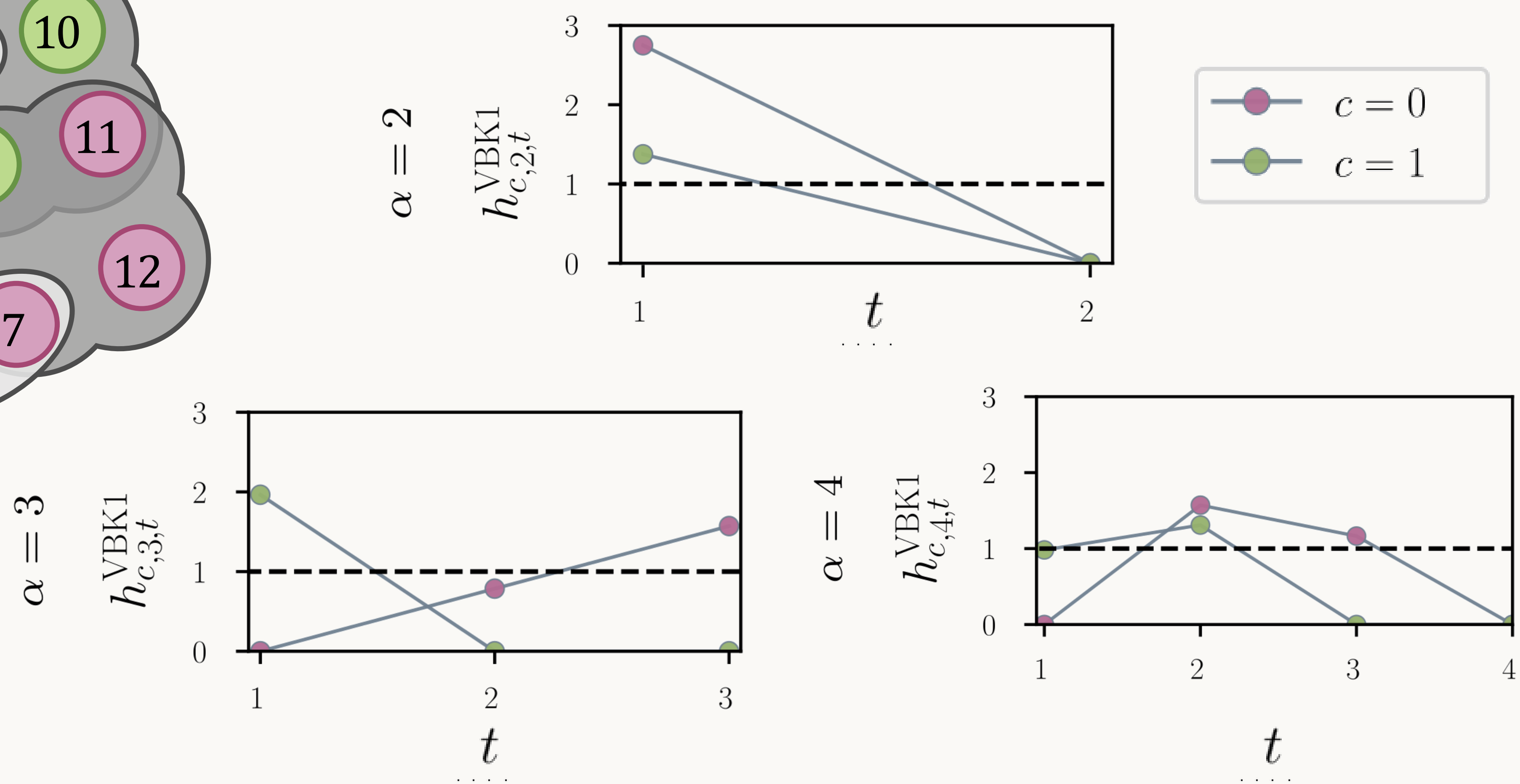

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Example Model: H3 Model

Nodes

1

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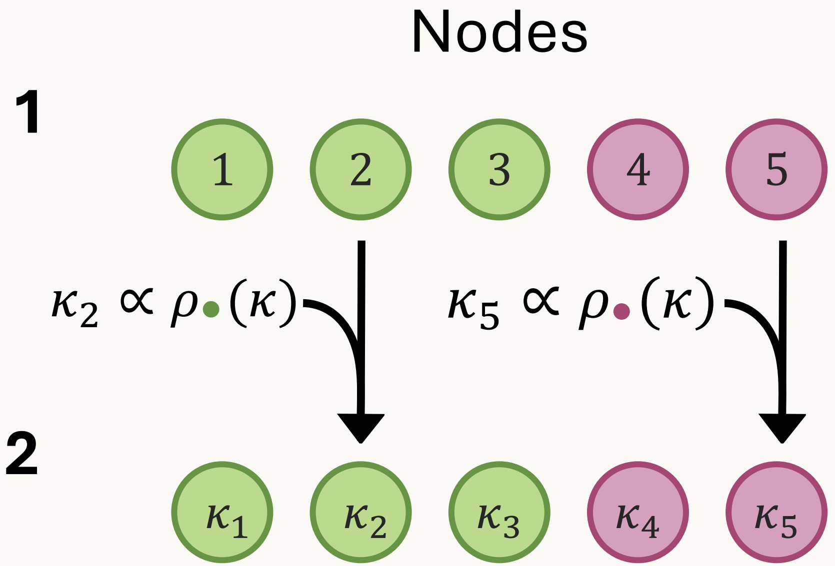

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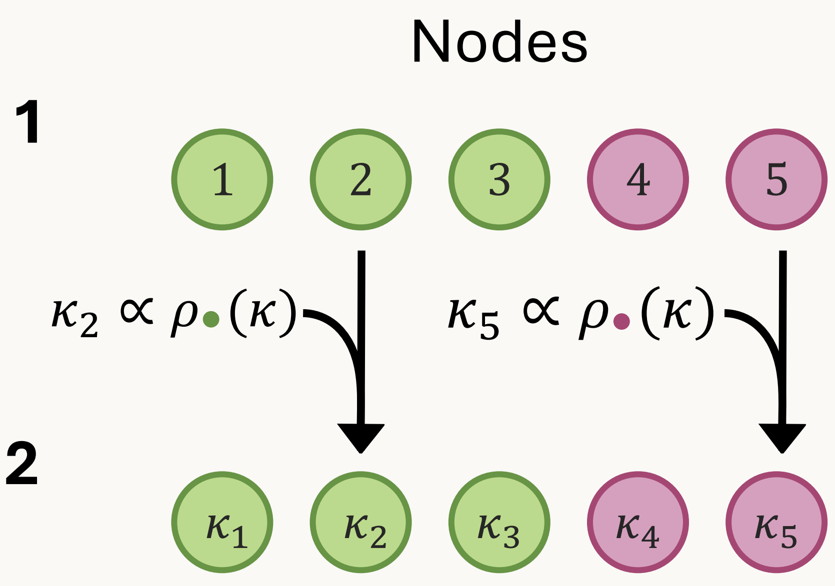

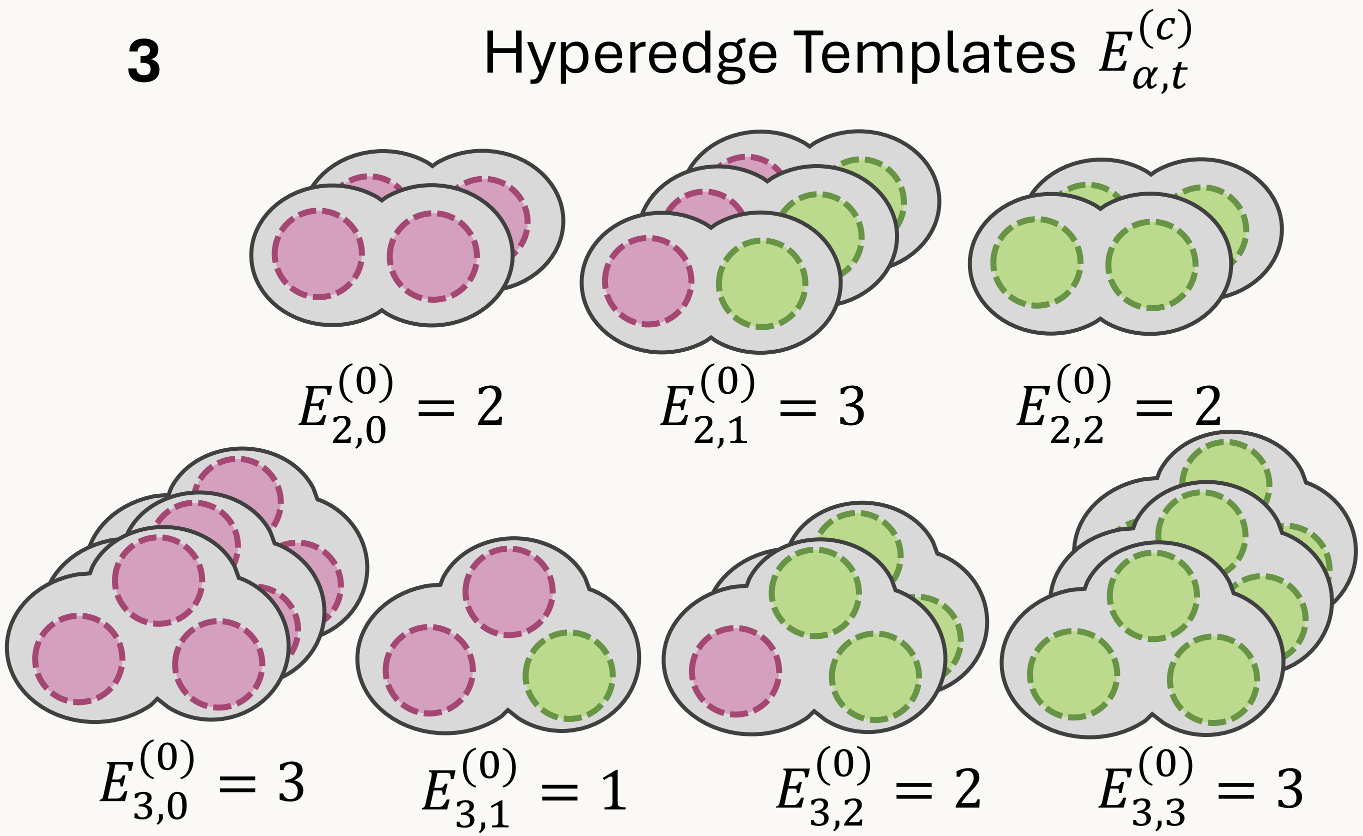

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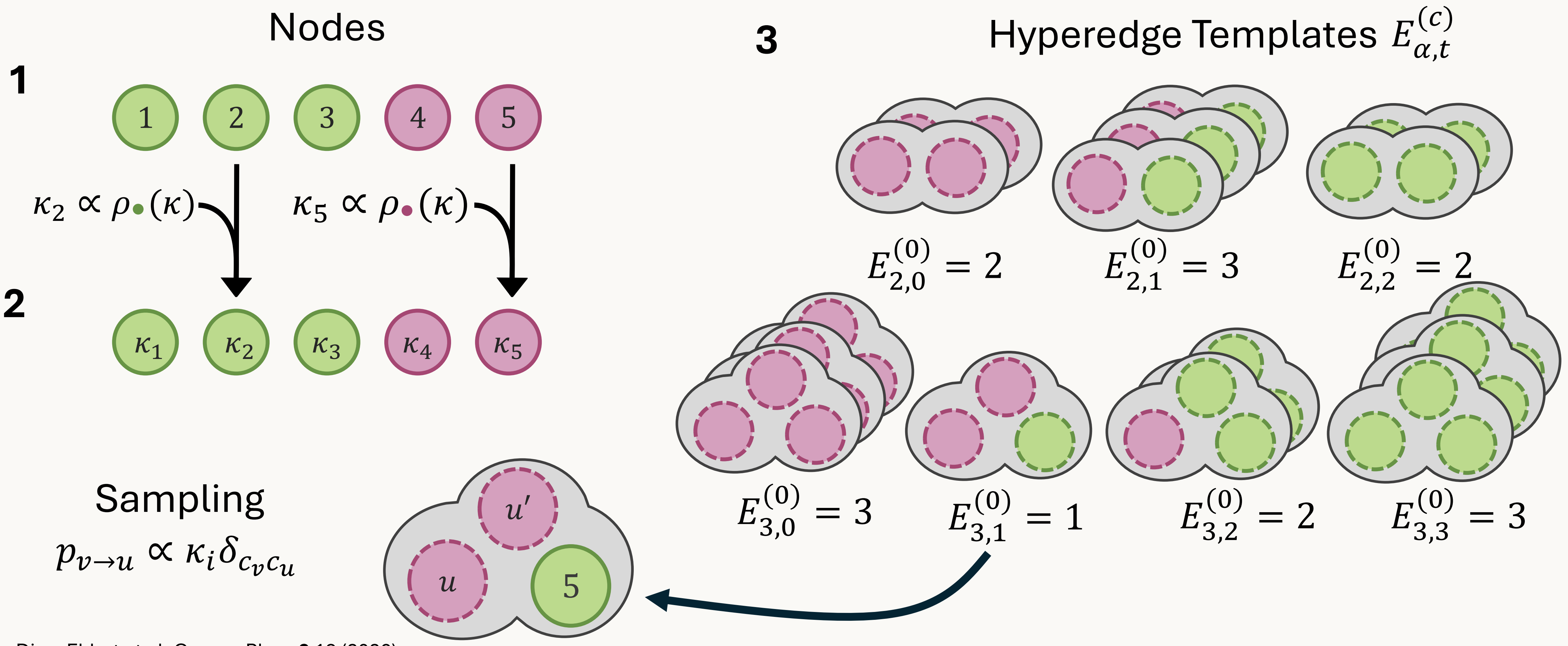

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Conclusion

Key Insights

- Hyperedges are **not either within- or between-class.** Higher-order mixing patterns are much richer.
- Mixing patterns can **vary with hyperedge size**.
- There is a **lot of methods** out there, but less effort to re-connect theory in the social sciences and a lack of synthesis.

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