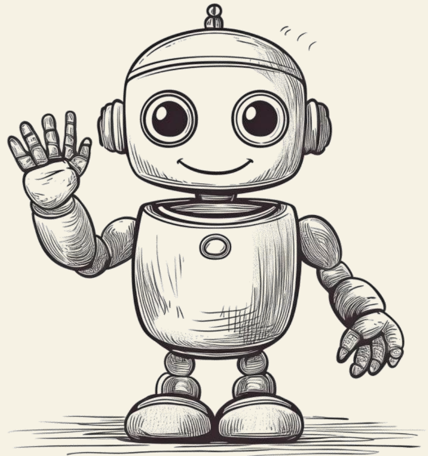
Neural ODE Tutorial

Can robots do differential equations?

Moritz Laber
Friday, 20th June 2025 at SFI CSSS2025

laber.m@northeastern.edu



The Good Old Days of Mathematical Modelling

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Can we find a quantitative, mathematical description of this phenomenon?

Of course! Let's use *differential equations*!

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Maki-Thompson Rumor Model

I **informed individuals**: know the rumor, spread it..
S **susceptible individuals**: don't know the rumor but…would like to.
R **removed individuals**: know the rumor, don't care to spread it.

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \beta is - \mu i^2 - \mu ir$$

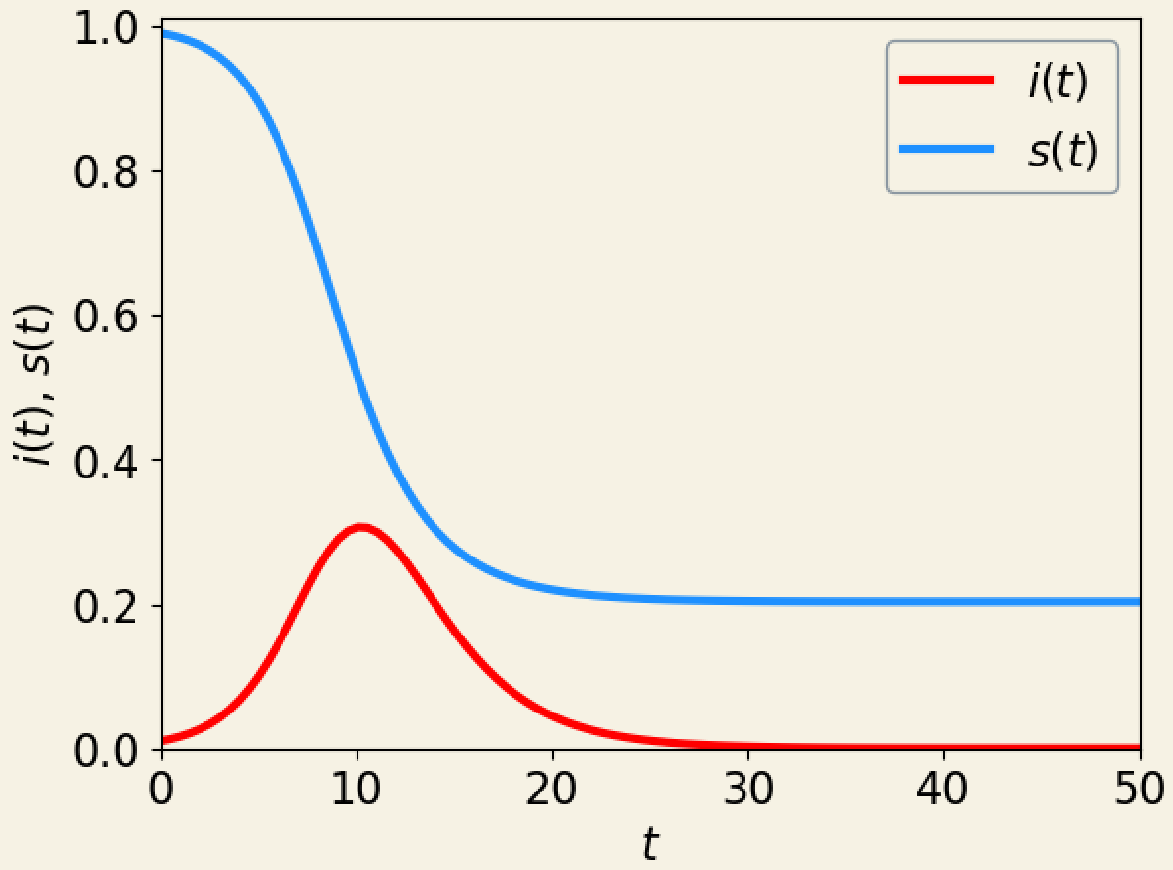
$$\frac{\mathrm{d}s}{\mathrm{d}t} = -\beta is$$

$$r = 1 - s - i$$

The Good Old Days of Mathematical Modelling

Numerical Solution

- Fix an initial condition: $(i_0, s_0, 1 - s_0 - i_0)$.
- Integrate the differential equation



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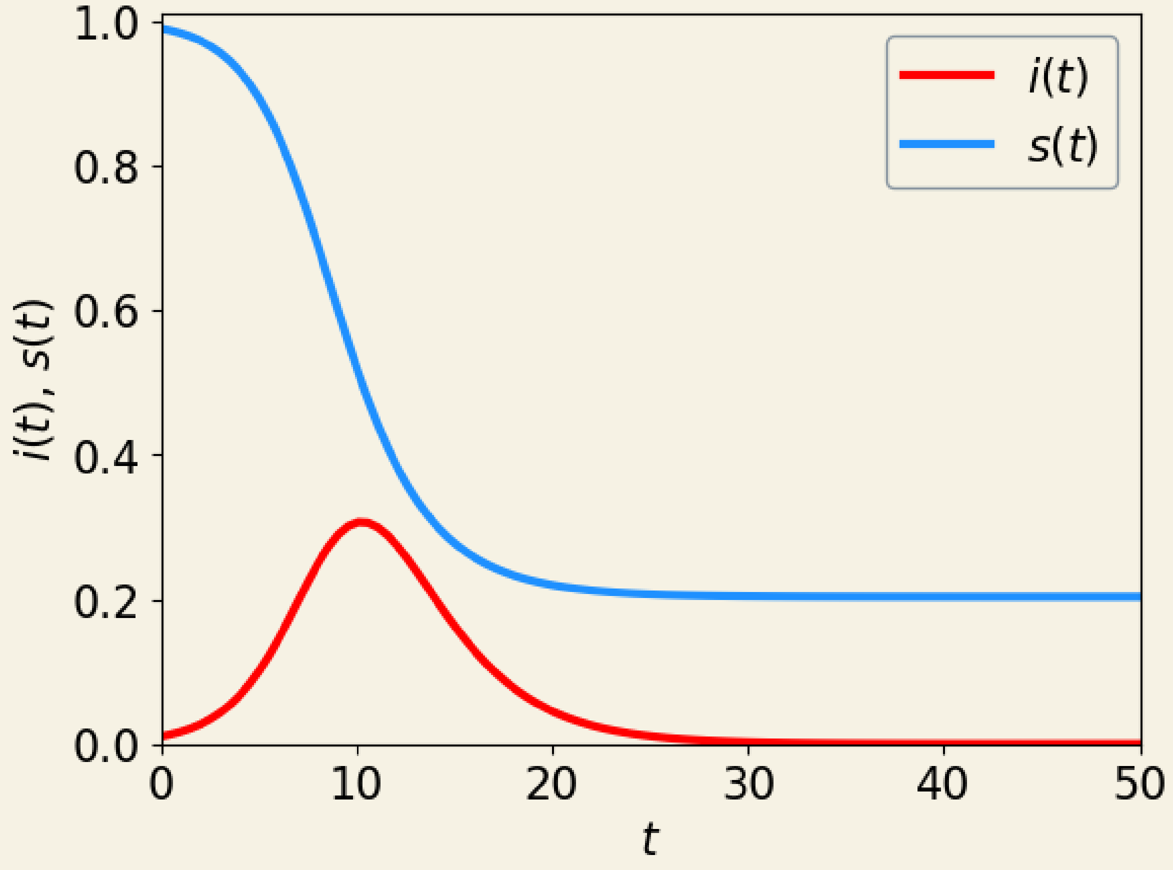
Analytical Treatment

- Fixed Points and Stability Analysis
- Steady States



A Modern, Data-Driven Twist on an Old Story

What did we do well?

- Interpretable: All terms have clear meaning.
- Robust: Captures main trends.

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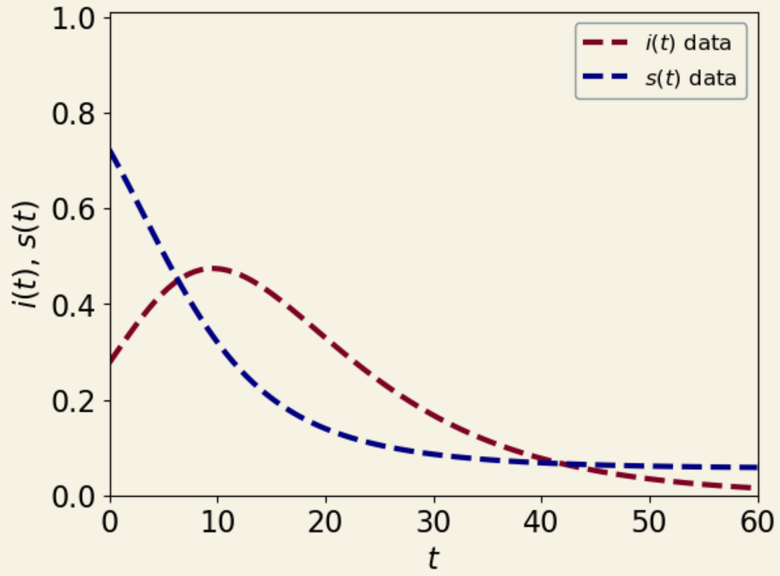

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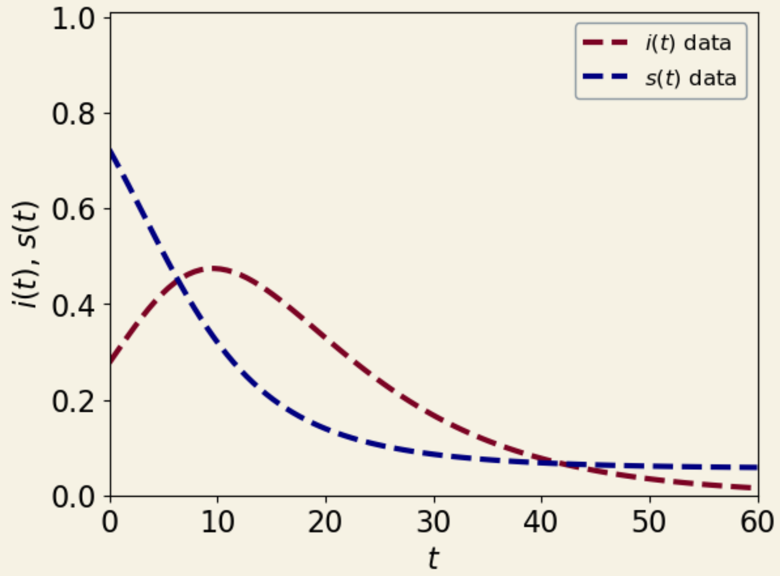

$$\frac{\mathrm{d}i}{\mathrm{d}t} = f_\theta(i, s)$$

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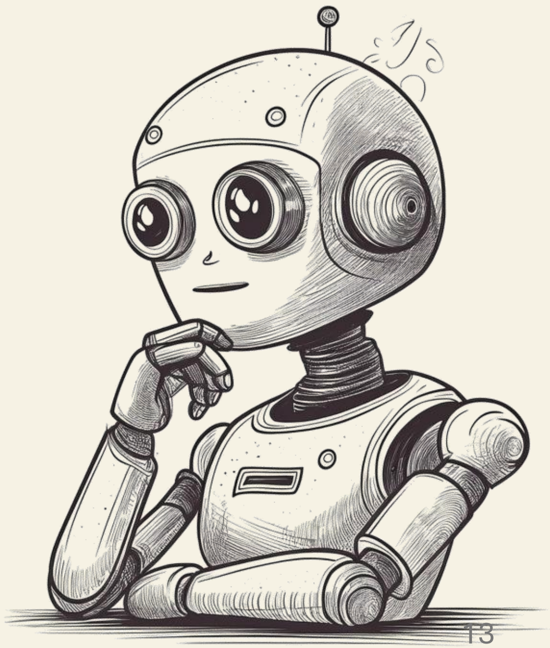

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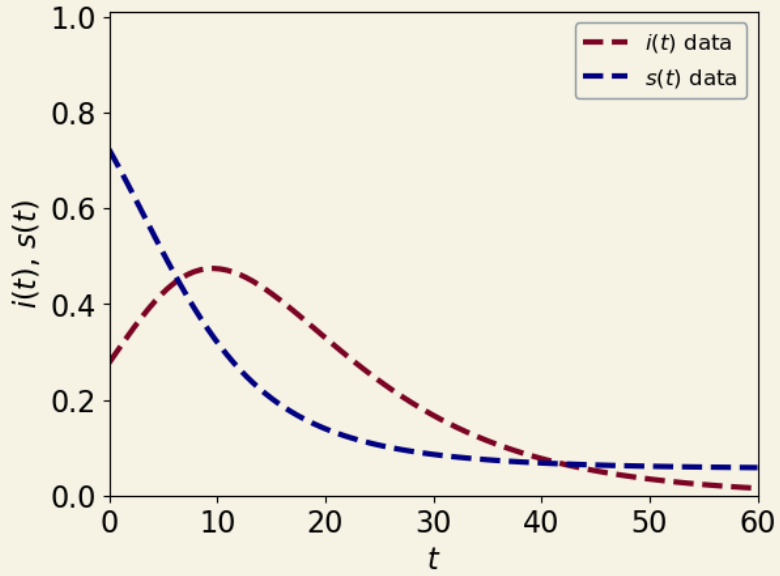

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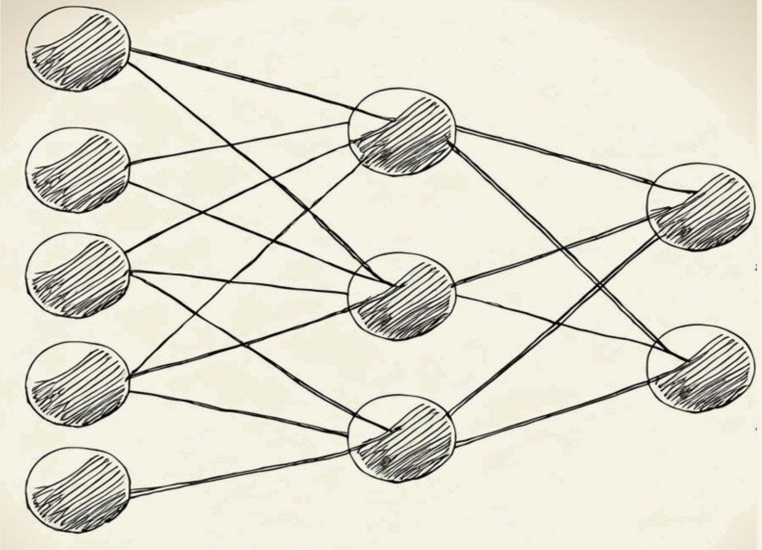
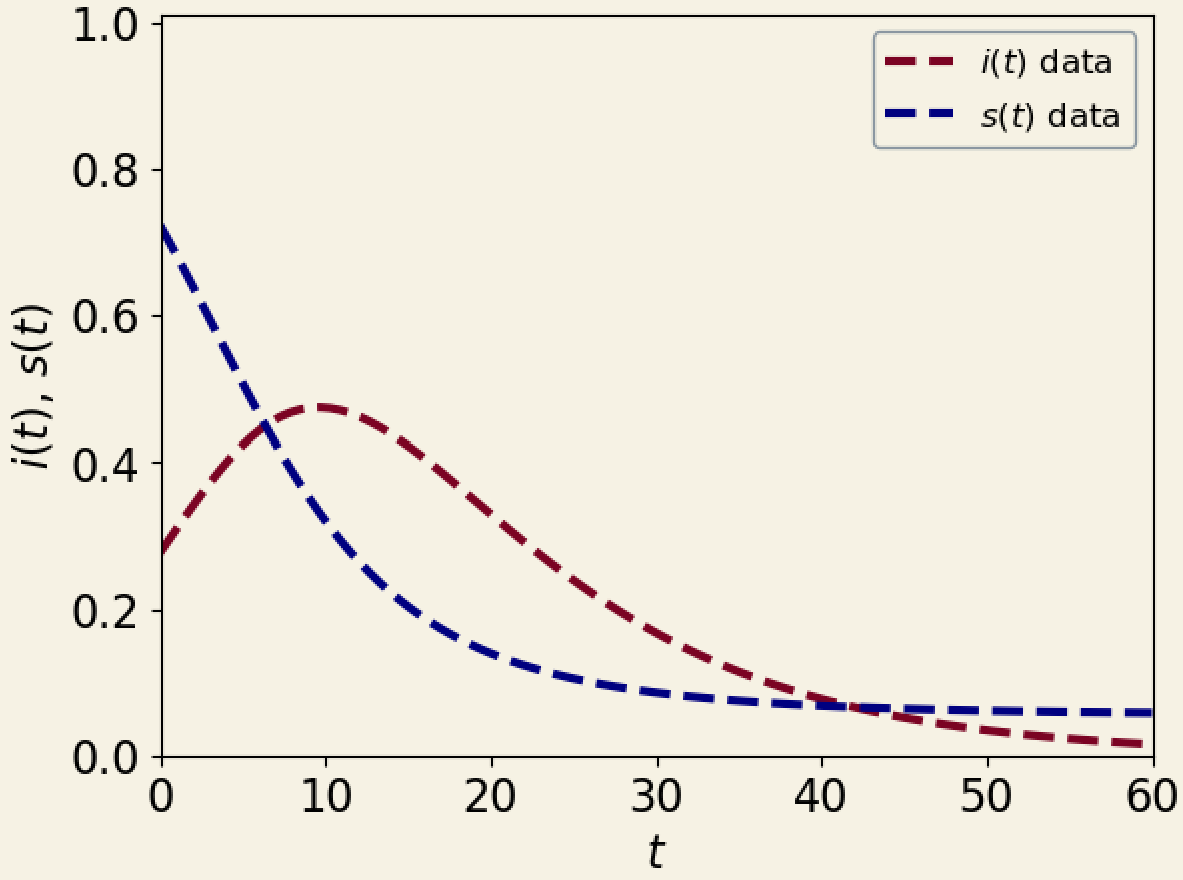


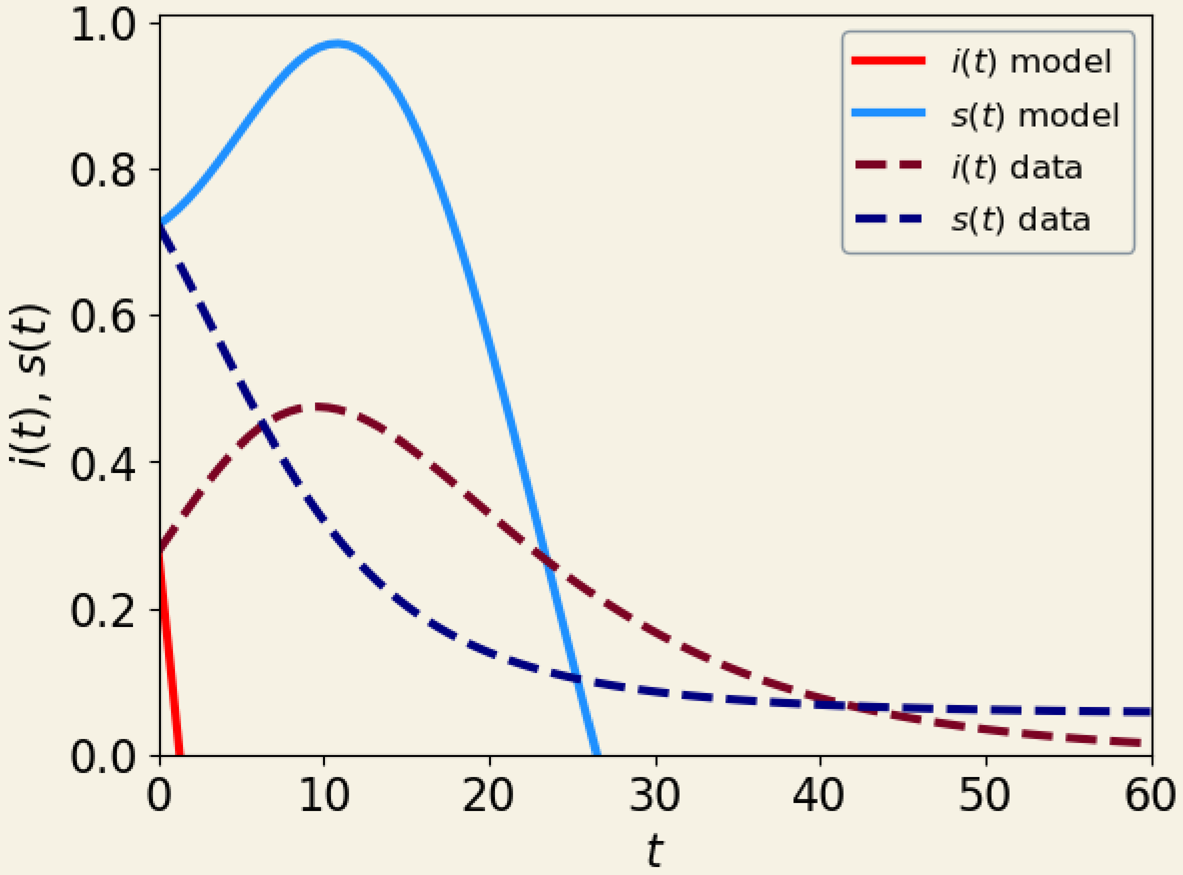
Training Neural ODEs: Broad Strokes



1. Gather Data of the System, usually starting from different initial conditions.

Training Neural ODEs: Broad Strokes

$$(\hat{s}, \hat{\imath})(t) = \texttt{ODEInt}[f_\theta, g_\theta]$$



1. Gather Data of the System, usually starting from different initial conditions.

2. Forward Pass: Solve an ODE with a neural network as right-hand side numerically.

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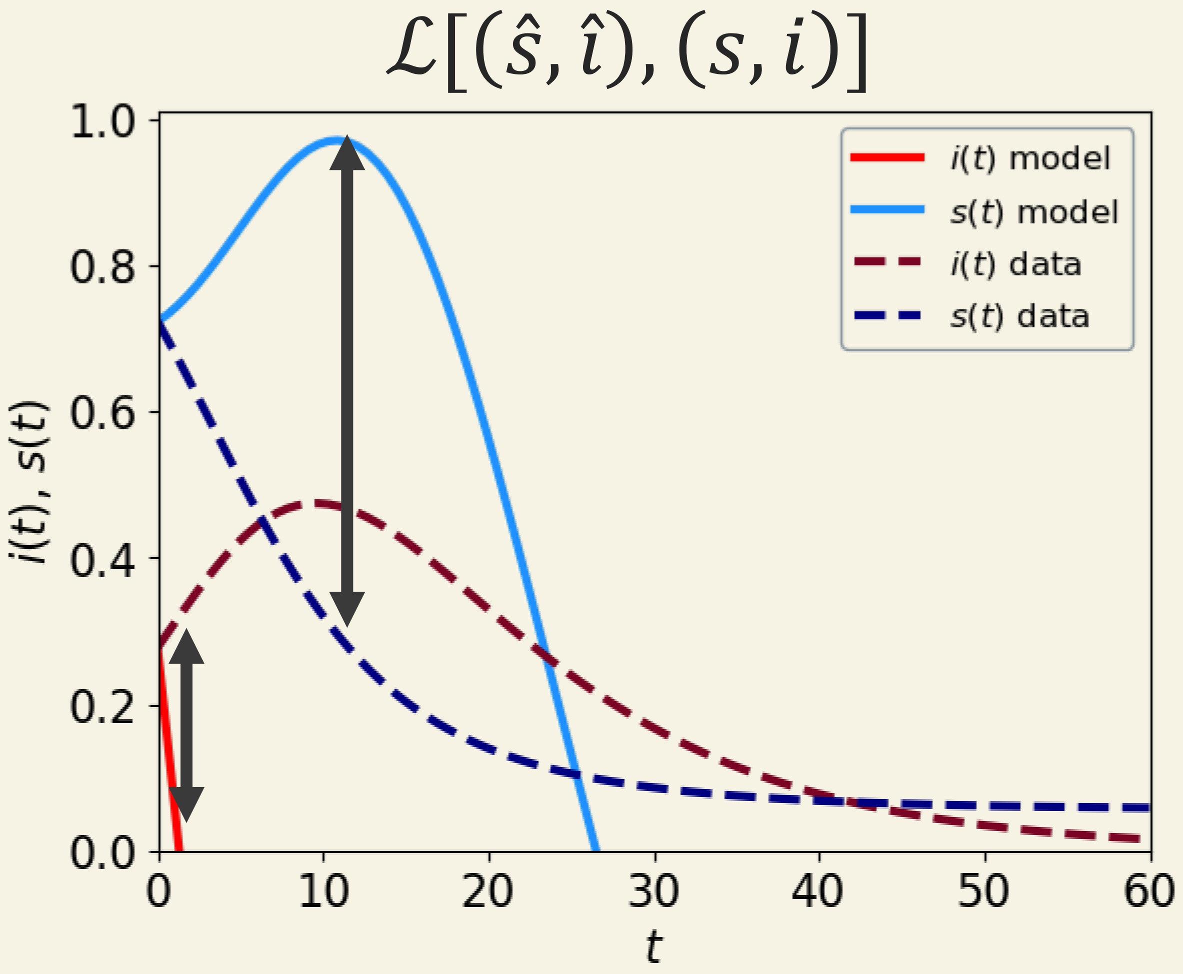

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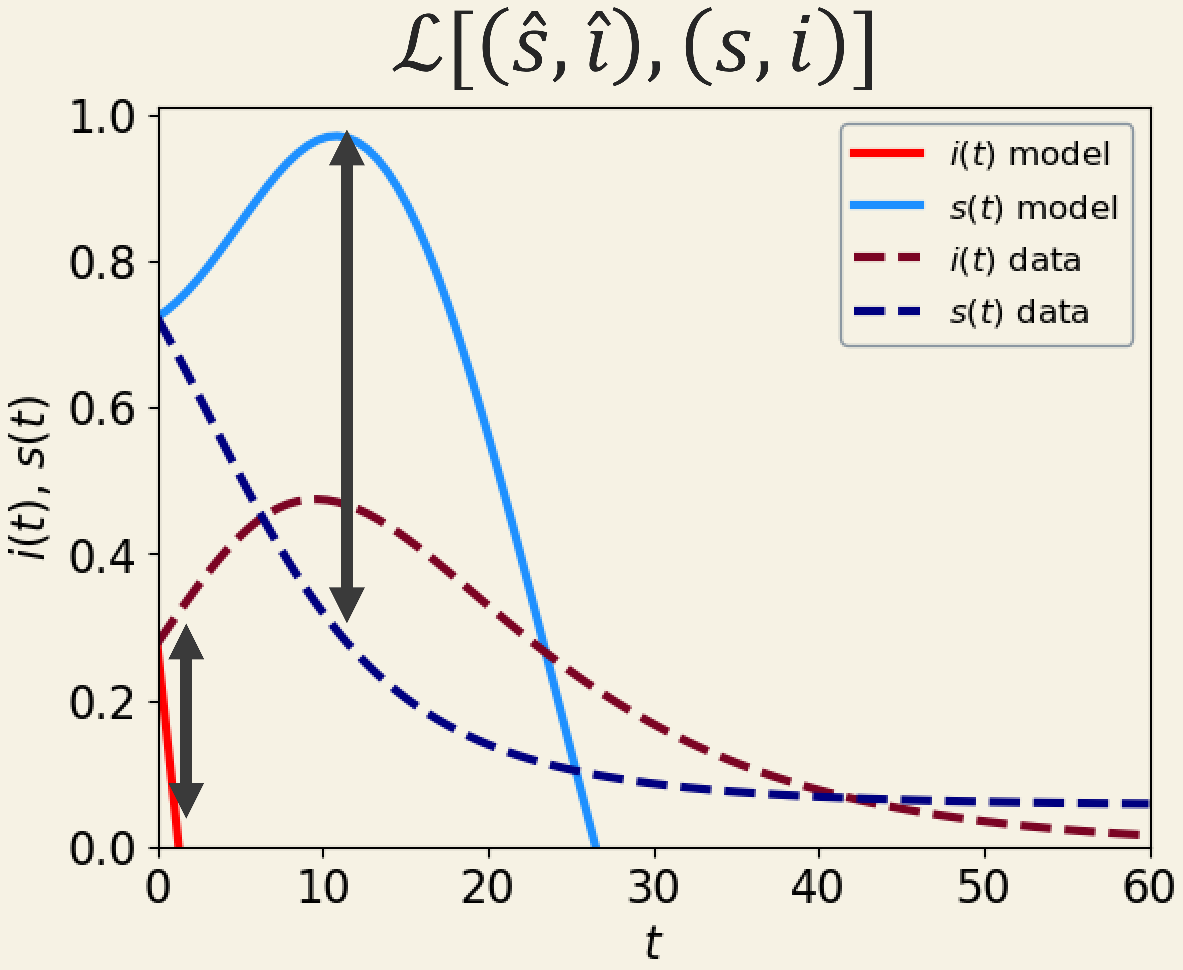

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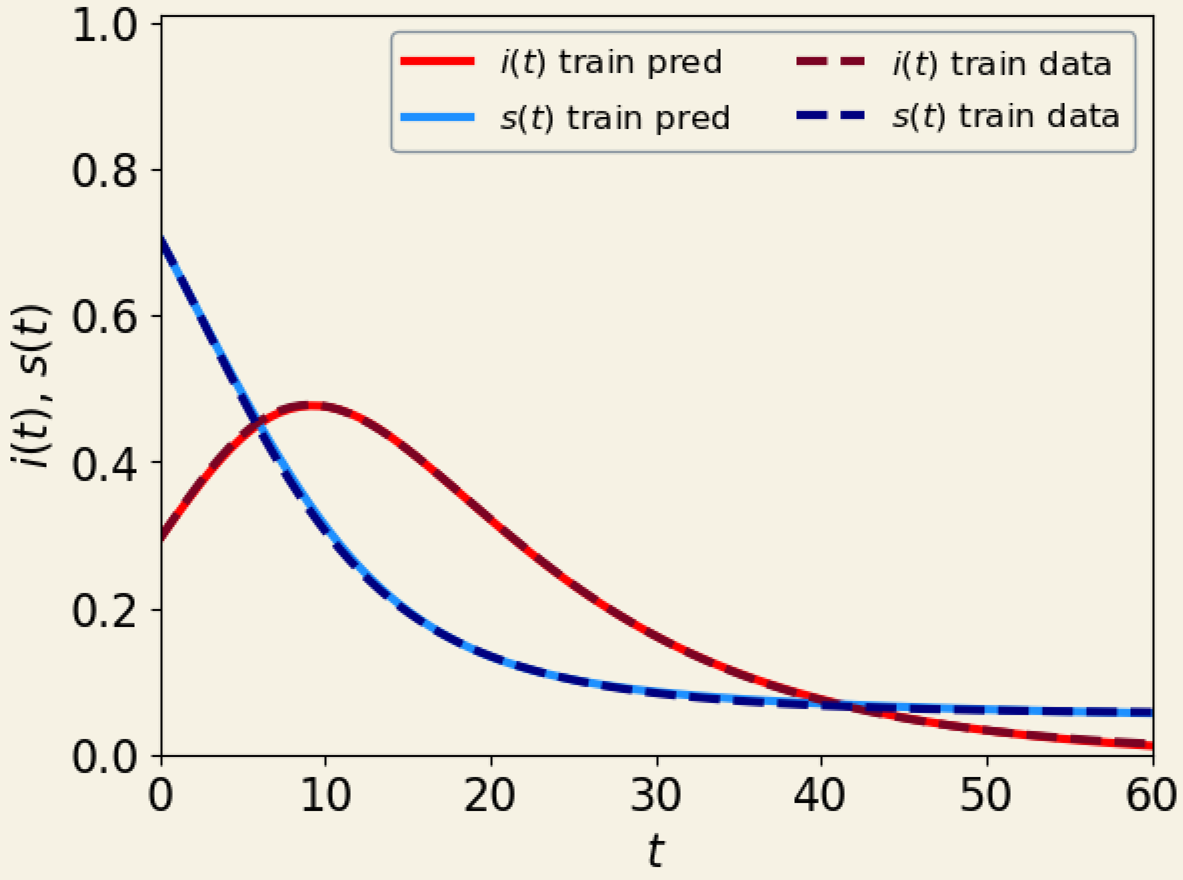
$$\theta \leftarrow \theta + \nabla_\theta \mathcal{L}[(\hat{s}, \hat{\imath}), (s, i)]$$



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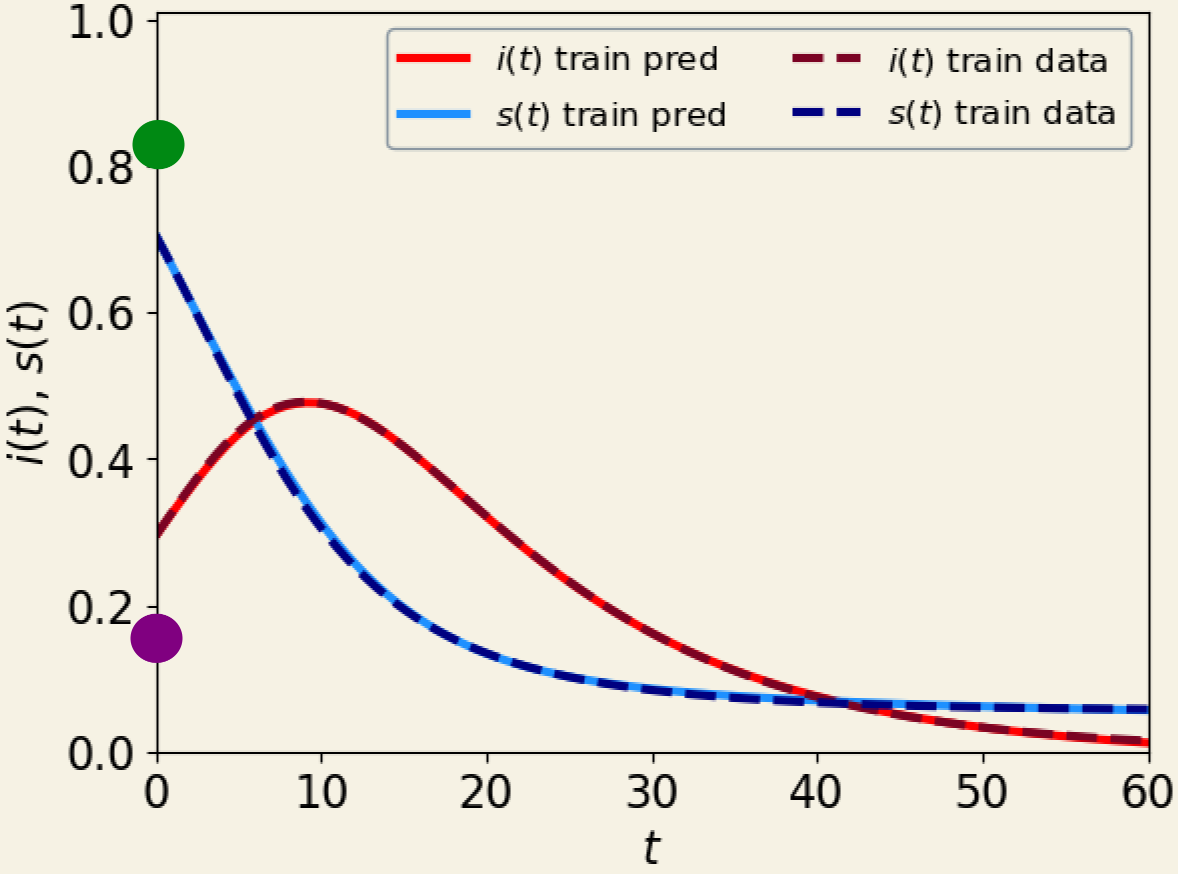
2. Forward Pass: Solve an ODE with a neural network as right-hand side numerically.

3. Compute the loss, e.g., RMSE.

4. Backward Pass: Compute gradients by backpropagation through the ODE solver, and update neural network weights.

Application 1: Prediction

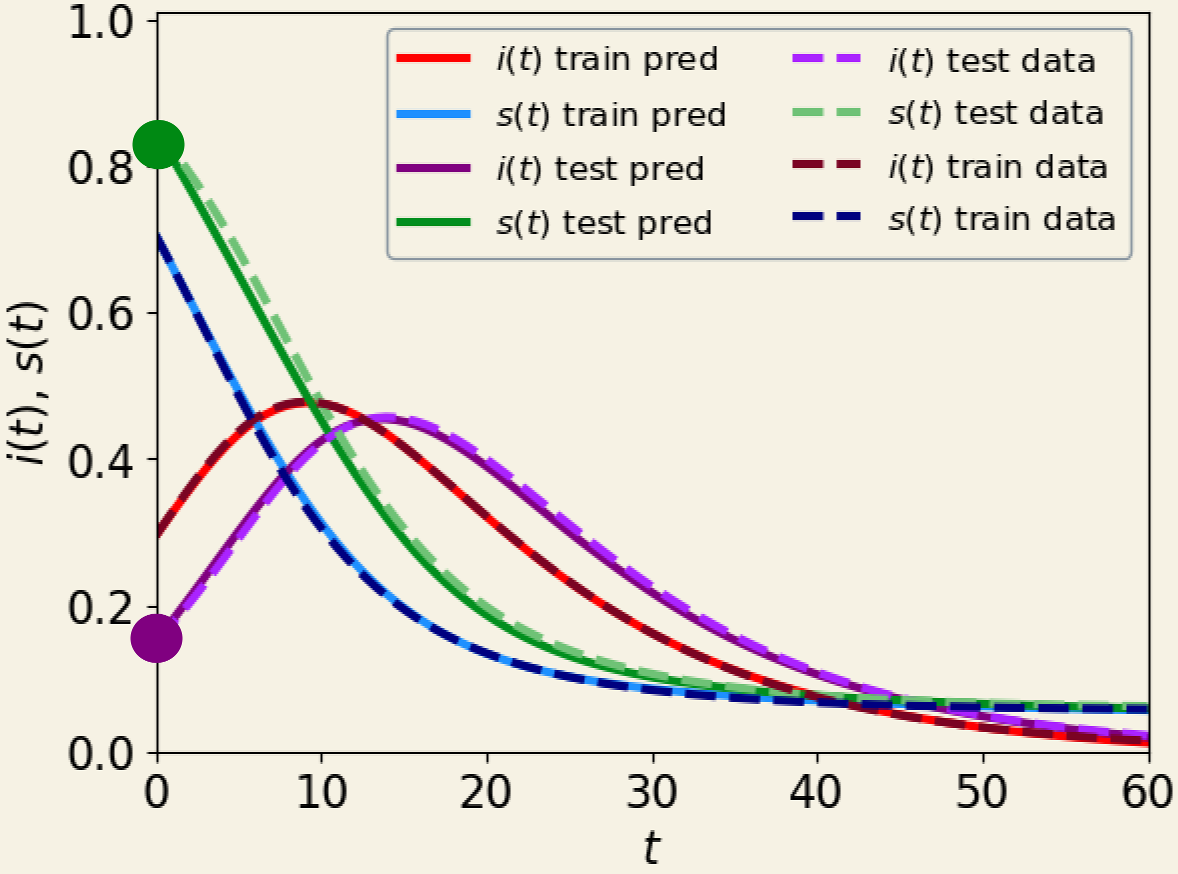
Question: Given the initial condition $i(t=0), s(t=0)$, how will the system evolve?



Application 1: Prediction

Question: Given the initial condition $i(t=0), s(t=0)$, how will the system evolve?

Solution: Feed the initial condition into the neural ODE and use a numerical ODE solver to make a prediction.



Application 2: Combination with ODEs

Question: Can we model a system that we do not fully understand at the relevant scale?

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \beta i s - \ ??? \ - \mu i(1 - i - s)$$

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$$\frac{\mathrm{d}i}{\mathrm{d}t} = \beta i s - f_\theta(i) - \mu i(1 - i - s)$$

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Solution: You can combine analytical terms with learned terms to arrive at more accurate descriptions of the system.

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Question: Can we find the analytical form of an ODE that describes the system?

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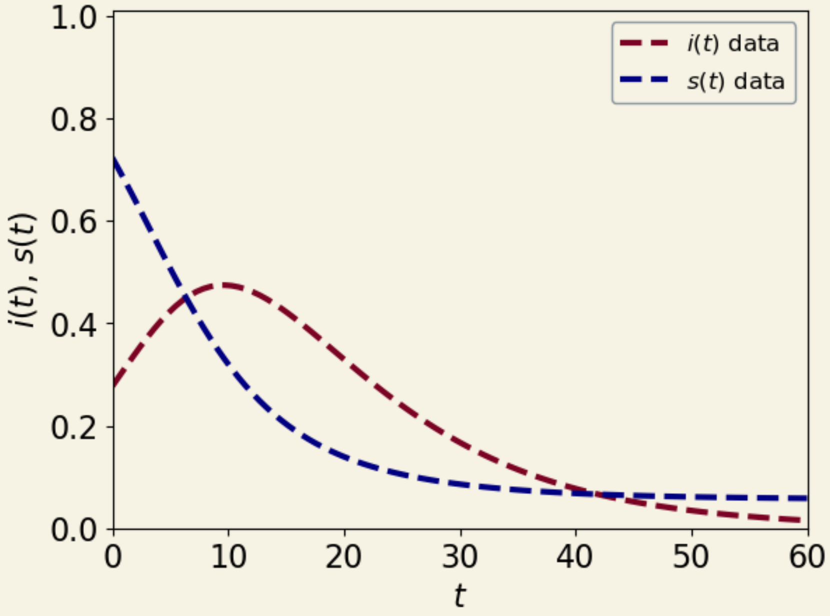

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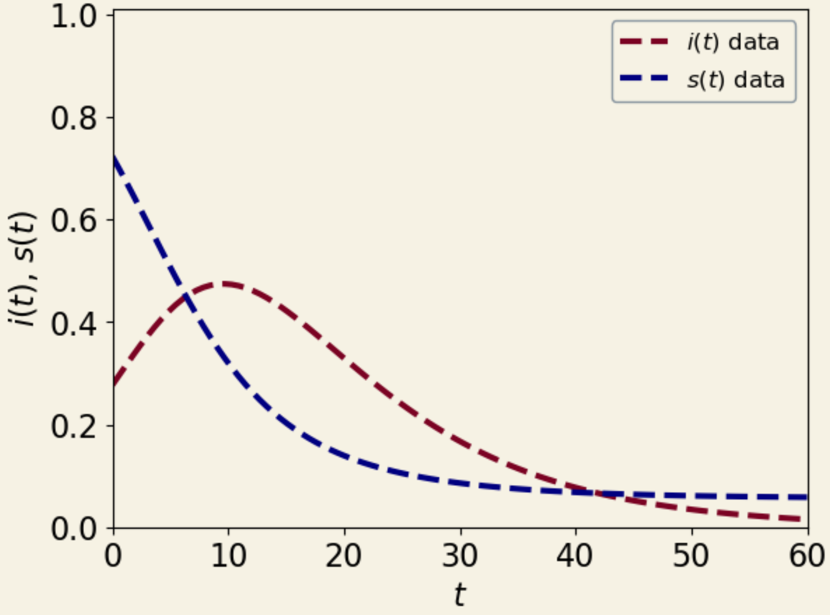

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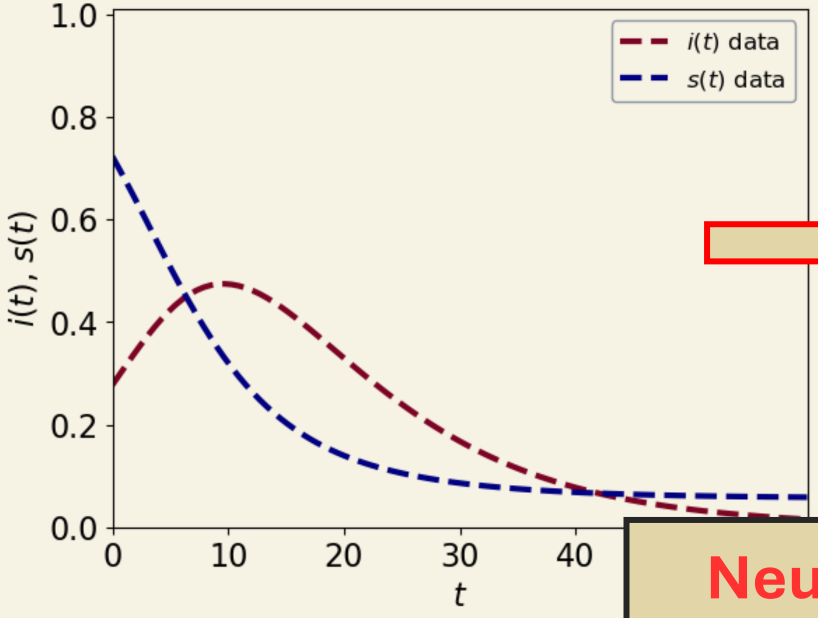

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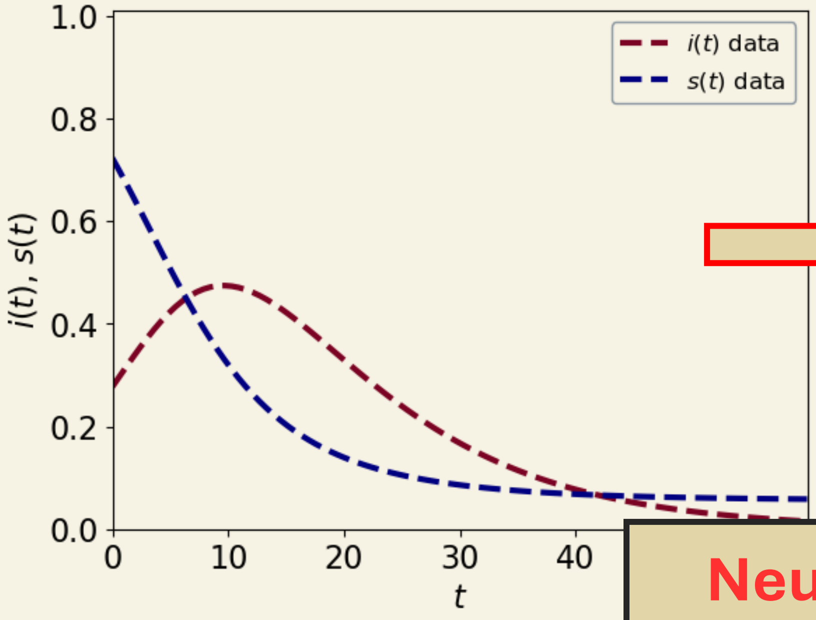

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Symbolic Regression

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Shameless self-plug: Soon, you'll find a paper on the topic on my google scholar. 😏

Hands On Neural ODEs

Intro to jax

[UvA DL Notebooks]

[jax tutorial]

Neural ODEs

[diffrax neural ODE tutorial]

[Uva DL Notebooks]

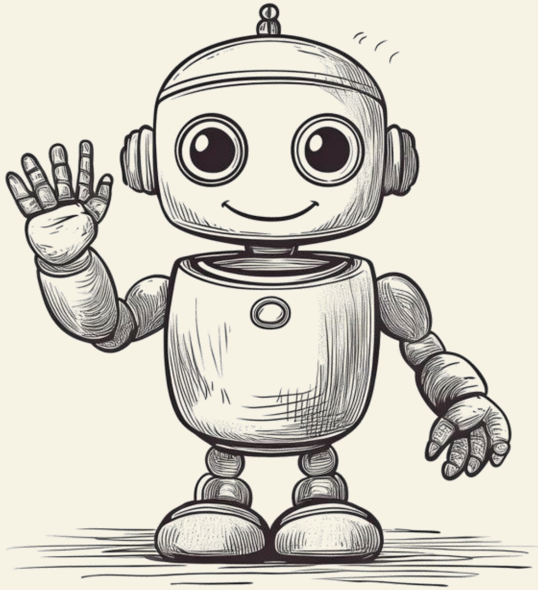
or klick

[here]

Other Tutorials

[diffrax tutorials]

Thank you.



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Laber M., Klein B., Eliassi-Rad T. *When do neural ordinary differential equations generalize on complex networks?* arXiv:2602.08980 (2026)